

# PEAK-LOAD PRICING IN A VERTICAL SETTING: THE CASE OF AIRPORTS AND AIRLINES

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## Abstract

Economists have long approached the airport congestion problem by calling for the use of the price mechanism, under which landing fees are based on a flight's contribution to congestion. In this paper we extend the existing work on airport congestion pricing, which does not consider inter-temporal pricing across different travel periods, to peak-load pricing (PLP) and analyze both price level and price structure (peak vs. non-peak). A major innovation of our analysis lies in the basic model structure used, in which an airport makes its capacity and price decisions prior to the airlines' decisions. This vertical structure gives rise to *sequential* PLP: the PLP schemes implemented by the downstream airlines induce a different periodic demand for the upstream airport, with the shape of that demand depending on the number of downstream carriers and the type of competition they exert. The airport then would have an incentive to use PLP as well, which in turn affects the downstream firms' PLP. We carry out the analysis for a public airport, for a private airport and for a private airport that has a strategic agreement with the airlines. The comparison between private and public airports is important because it has been argued that private airports would use efficient peak-load and congestion pricing. Our results show that private airports will not only have higher peak and off-peak prices (levels), but also have higher price differentials, inducing a quite different allocation of flights and passengers to peak and off-peak periods. Further, it may be possible that a public airport find it optimal to have a peak price that is lower than the off-peak price. Finally, we note that, while there is an extensive body of literature on PLP, the case of sequential peak-load pricing has yet been analyzed. Since this sequential structure is highly relevant to many other industries (such as telecommunications), our results not only contribute to the understanding of airport policy and management, but should be useful for other sectors as well.

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## INTRODUCTION

In the last several years airlines and passengers have been suffering from congestion at most of the major airports in the world, and air traffic delays have become a major public policy issue. Economists have approached airport congestion by calling for the use of the price mechanism, under which landing fees are based on a flight's contribution to congestion. Meanwhile, many airports around the world have recently been, or are in the process of being, privatized or corporatized. In Canada, for example, airports recently devolved from direct Federal control to become autonomous entities, and major airports are now managed by private not-for-profit (but subject to cost recovery) corporations. One of the leading arguments for airport privatization/corporatization is that private airports would use more efficient peak-load and congestion pricing schemes than public airports. This is an important issue because, while congestion pricing is desirable from an efficiency point of view, it has not really been implemented; the existing landing fees are based primarily on aircraft weight (Schank, 2005).

In this paper we extend the existing work on airport congestion pricing, which does not consider inter-temporal pricing across different travel periods, to peak-load pricing (PLP) and analyze both the price level and price structure (peak vs. non-peak). A major innovation of our analysis lies in the basic model structure used, which has strong implications for PLP. Here, an airport makes its capacity and price decisions prior to the airlines' output decisions. This vertical structure gives rise to *sequential* PLP: the PLP schemes implemented by the downstream airlines induce a different periodic demand for the upstream airport, with the shape of that demand depending on the number of downstream carriers and the type of competition they exert. The airport then would have an incentive to use PLP as well, which in turn affects the way the downstream firms use PLP.

We carry out the analysis for a public airport that maximizes social welfare, for a private, unregulated profit-maximizing airport, and for a private airport that has some sort of strategic agreement with the airlines using it. The comparison of the three cases then allows us to shed some light on the claim that private airports would use more efficient congestion and peak-load pricing than public airports, which may have important implications for policy. Our main results indicate that private airports will always use PLP, and that they will not only have higher peak and off-peak prices (levels), but also have the highest price differential, which would induce a quite different allocation of flights and passengers to the peak and off-peak periods. On the other hand, it may be possible that a public airport find it optimal to have a peak price that is lower than the off-peak price. Finally, an airport with a strategic agreement with the airlines may not use PLP; further, its pricing practices would induce in the airline market the same outcome as absence of downstream competition.

There is an extensive body of literature on peak-load pricing. The classical papers on peak-load pricing (Boiteaux 1949, 1960; Steiner 1957; Hirschleifer 1958; Williamson 1966) focused on normative rules for pricing a public utility's non-storable service subject to periodic demands. The usual assumptions were: (i) demand is constant within each pricing period; (ii) demand in one period is independent of demand in other periods; (iii) constant marginal costs; (iv) the length of pricing periods is exogenous; and (v) the number of pricing periods is exogenous. Many authors have since contributed to the generalization of PLP

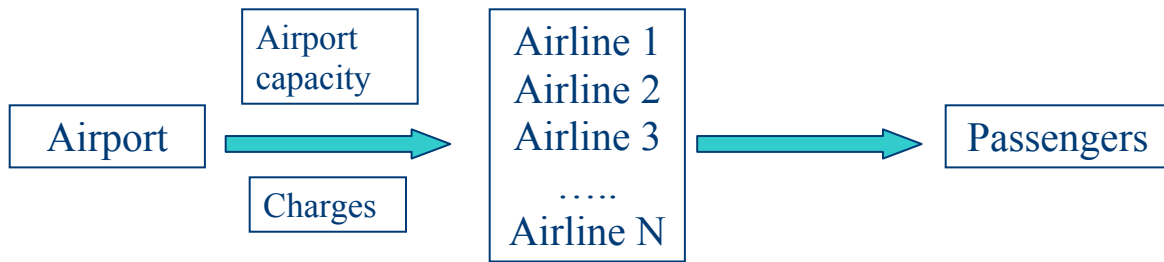
results by relaxing one or a group of these assumptions, including Pressman (1970), Panzar (1976), Dansby (1978), Craven (1971, 1985), Crew and Kleindorfer (1986, 1991), Gersten (1986), De Palma and Lindsey (1998), Laffont and Tirole (2000), Shy (2001) and Calzada (2003). However, the case of sequential peak-load pricing, be it for public or private utilities, has yet been analyzed. In the telecommunications research, for instance, Laffont and Tirole looked at PLP only at the upstream level (the network access charge) whilst Calzada considered PLP only at the downstream level.

In the airport research, three empirical models of congestion pricing have been developed, namely the standard PLP model (Morrison 1983; Morrison and Winston 1989), the deterministic bottleneck model (Vickrey 1969; Arnott et al. 1993), and the stochastic bottleneck model (Daniel 1995, 2001). These studies considered PLP only at the airport level. Brueckner (2002) and Barbot (2004), on the other hand, investigated PLP only at the airline level. Further, most of these studies considered a public airport that maximizes social welfare, and none made an assessment of the effects of privatization on price structures.

The paper is organized as follows. In the next Section we set out the basic model. We then analyze and characterize the output market equilibrium paying particular attention to the peak and off-peak derived demands for airport services. We finally examine the airport's decisions and describe how the airport ownership and airline-airport relationship influence the peak and off-peak prices and welfare. The last Section contains concluding remarks.

## THE MODEL

Our basic framework follows Brueckner (2002) by employing a two-stage model of airline and airport behavior. There are  $N$  homogeneous air carriers servicing a congestible airport. In the first stage the airport decides on the airport charge  $P$  and capacity  $K$ , where  $K$  is continuously adjustable. In the second stage, each carrier chooses its output in terms of the number of flights. The situation is depicted in Figure 1.



**Figure 1:** Timing of the game analyzed

There is a continuum of consumers (passengers) labelled by  $\theta$  and distributed uniformly on  $\Theta = [\underline{\theta}, \bar{\theta}]$ . We normalize the number of total consumers to  $\bar{\theta} - \underline{\theta}$  so the number of passengers with type belonging to  $[\theta_1, \theta_2]$  is directly given by  $\theta_2 - \theta_1$ . Consumers' utility function may be written as  $W(x, B_h(\theta), -D_h)$ , where  $x$  is a vector of products,  $B_h(\theta)$  denotes

–for consumer  $\theta$ – (gross) benefits if he travels in period  $h$  ( $h=p$ , peak period;  $h=o$ , off-peak period;  $h=n$ , not traveling), and  $D_h$  denotes the flight delay associated with travel in period  $h$ . Note that we have the three cases, namely, peak, off-peak and non-travelling, in the sense that  $B_p(\theta) > B_o(\theta) > B_n(\theta) = 0$ . This means that, on one hand, if travel was free and without congestion, the consumer would always prefer traveling to non-traveling. On the other hand, with identical airfares and delays, consumers would always prefer traveling in the peak period to traveling in the off-peak period. Thus, peak and off-peak periods are vertically differentiated.

As is usual in discrete choice models, we solve consumers' optimization problem in two steps:

$$\begin{aligned} & \max_x \left\{ \max_{h \in p, o, n} W(x, B_h(\theta), -D_h) \right\} \\ \text{s.t.} \quad & P_x \cdot x + t_h \leq I(\theta) \end{aligned} \quad (1)$$

where  $I(\theta)$  is income for consumer  $\theta$  and  $t_h$  is the airfare in period  $h$ . The first maximization leads to the conditional Marshallian demands,  $x^*(P_x, I(\theta) - t_h, B_h(\theta), -D_h)$ . Replacing these in  $W$  we obtain,  $W(x^*, B_h(\theta), -D_h) \equiv V_h(P_x, I(\theta) - t_h, B_h(\theta), -D_h)$ . This is the conditional indirect utility function. For simplicity, we assume that  $V_h$  is linear and  $B_h(\theta)$  takes the simple form  $\theta B_h$ , with  $B_h$  being constant. This then leads to:

$$V_h = \psi \cdot P_x + \lambda(I(\theta) - t_h) + \theta B_h - \alpha D_h \quad (2)$$

For the second maximization –comparisons of  $V_h$  for different  $h$ – we focus on the elements that determine the discrete choice, obtaining a truncated conditional indirect utility function (note that  $\psi \cdot P_x + \lambda I(\theta)$  plays no role). Dividing by the marginal utility of income,  $\lambda$ , and redefining  $\theta$  and  $\alpha$  (which becomes the subjective value of time savings) we then obtain:

$$\bar{V}_h(\theta) = \theta B_h - \alpha D_h - t_h \quad (3)$$

The demand problem we have set is identical to the one that will result if we fix  $\theta$  but allow the value of time  $\alpha$  to have a distribution among consumers. One could also argue that  $\theta$  and  $\alpha$  are related (Yuen and Zhang, 2005), but we do not do this here. In (3), the flight delay at period  $h$  is given by:

$$D_h = D(Q_h, K) = \frac{Q_h / L_h}{K(K - Q_h / L_h)} \quad (4)$$

where  $Q_h$  is the total number of flights in the period,  $L_h$  is the length of the pricing period, and  $K$  is the airport's capacity (measured in flights per hour). The functional form (4) was previously estimated from steady-state queuing theory (see Lave and DeSalvo 1968; U.S. Federal Aviation Administration 1969; Horonjeff and McKelvey, 1983; and Morrison 1987). Alternatively, a linear delay function,  $D(Q_h, K) = \delta Q_h / K$ , could be used (see e.g. Pels and Verhoef, 2004).

We first note the following characteristics about the allocations of consumers to periods: (i) if consumer  $\theta_1$  flies, then consumers  $\theta \geq \theta_1$  fly; (ii) if consumer  $\theta_1$  does not fly, then consumers  $\theta < \theta_1$  do not fly; and (iii) if  $\theta^*$  is indifferent between peak and off-peak traveling, then consumers  $\theta \geq \theta^*$  choose the peak and consumers  $\theta < \theta^*$  choose the off-peak.<sup>1</sup> Hence, if we denote  $\theta^f$  the consumer who is indifferent between flying and not flying and  $\theta^*$  the consumer who is indifferent between peak and off-peak, (i), (ii) and (iii) above show that, in the case of an interior solution, we get  $\underline{\theta} < \theta^f < \theta^* < \bar{\theta}$ . We assume interior allocations for now but later shall find conditions on the parameters for this to be the case. Notice further that, if the value of time was the one with a distribution, the results would be reversed in that low-values  $\alpha$  would travel in the peak, intermediate-values  $\alpha$  would travel in the off-peak and high-values  $\alpha$  would not travel. This is so because people with high value of time, *ceteris paribus*, are the first ones to move towards the less congested period and, if congestion is still too high there, to decide not to fly.

To obtain the consumer demands for peak and off-peak in terms of flights, we need assumptions about the length of the pricing periods, aircraft size and load factors. We assume: (i) Fixed proportions:  $S = \text{Aircraft Size} \times \text{Load Factor}$ , is constant and the same across carriers; (ii)  $L_h$ , the length of the pricing periods, is fixed and exogenously given, and is the same for airlines; (iii) Passengers are uniformly allocated within each pricing period, i.e. there is no intra-period demand fluctuation; and (iv)  $L_o$  is long enough so that  $D(Q_o, K) = 0$  throughout. Given these assumptions, the demands for the peak and off-peak periods are, respectively,  $q_p = Q_p S = \bar{\theta} - \theta^*$  and  $q_o = Q_o S = \theta^* - \theta^f$ , where  $q_h$  is the total number of passengers in period  $h$ . Thus

$$\begin{aligned}\theta^* &= \bar{\theta} - Q_p S \\ \theta^f &= \theta^* - Q_o S\end{aligned}\tag{5}$$

The final flyer is determined by  $\theta^f B_o - t_o = 0$ , whereas the indifferent flyer is determined by  $\theta^*(B_p - B_o) - \alpha D_p - (t_p - t_o) = 0$ . From these and (5) we get:

$$t_o(Q_o, Q_p) = B_o \bar{\theta} - B_o S Q_o - B_o S Q_p\tag{6}$$

$$t_p(Q_o, Q_p) = B_p \bar{\theta} - B_o S Q_o - B_p S Q_p - \alpha D(Q_p, K)\tag{7}$$

Equation (6) is the inverse demand function faced by the airlines for the off-peak period, whereas (7) is the inverse demand function for the peak period. Note that the demand functions are not linear if  $D$  is not. Further, analytical expressions of the cross elasticity of demand between peak and off-peak periods can be obtained.

We now turn to the airlines. They have identical cost functions, given by:

$$C_A^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h, K) = \sum_{h \in p, o} Q_h^i [c + P_h + \beta D(Q_h, K)]\tag{8}$$

where  $Q_h^i$  is the number of airline  $i$ 's flights in period  $h$ ,  $P_h$  is the airport charge in period  $h$  and  $\beta$  is the airline's extra costs due to congestion. Airlines' profit functions are:

$$\phi^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h, K) = \sum_{h \in p, o} t_h(Q_o, Q_p) Q_h^i S - C_A^i(Q_h^i, \mathbf{Q}_h^{-i}, P_h, K) \quad (9)$$

With these functions at hand we have a well-defined airline sub-game, which we analyze and characterize in the next Section.

## ANALYSIS OF THE OUTPUT-MARKET EQUILIBRIUM

To solve for the sub-game perfect Nash equilibrium, we start with the analysis in stage 2. Given the airport's decisions on capacity and prices, airlines' first-order conditions are given by  $\partial \phi^i / \partial Q_h^i = 0$ . Calculating this, imposing symmetry, i.e.  $Q_h^i = Q_h / N$ , and re-arranging, we get:

$$\Omega^o(Q_o, Q_p, K, P_o, N) = (B_o \bar{\theta} S - c - P_o) - Q_o \frac{B_o S^2 (N+1)}{N} - Q_p \frac{B_o S^2 (N+1)}{N} = 0 \quad (10)$$

$$\begin{aligned} \Omega^p(Q_o, Q_p, K, P_p, N) &= (B_p \bar{\theta} S - c - P_p) - Q_o \frac{B_o S^2 (N+1)}{N} - Q_p \frac{B_p S^2 (N+1)}{N} \\ &\quad - (\alpha S + \beta) \left( D(Q_p, K) + \frac{Q_p}{N} D_Q(Q_p, K) \right) = 0 \end{aligned} \quad (11)$$

where  $D_Q \equiv \partial D / \partial Q$ . A useful equation that is easily obtained from (10) and (11) is:

$$\begin{aligned} -\Omega^p + \Omega^o &= Q_p \frac{(B_p - B_o) S^2 (N+1)}{N} + (\alpha S + \beta) \left( D(Q_p, K) + \frac{Q_p}{N} D_Q(Q_p, K) \right) \\ &\quad + (P_p - P_o) - \bar{\theta} S (B_p - B_o) = 0 \end{aligned} \quad (12)$$

Since (12) depends only on  $Q_p$ , it implicitly defines  $Q_p(P_o, P_p, K, N)$  which is the airport's demand for the peak period. Then,  $Q_o(P_o, P_p, K, N)$ , the airport's demand for the off-peak period, is obtained from (10). Given the airport demand functions, an analytical expression of the cross elasticity of demand between peak and off-peak periods can be obtained.

To characterize the output-market equilibrium, three questions naturally arise: (A) What are the conditions on the parameters that guarantee interior solutions, that is  $\underline{\theta} < \theta^f < \theta^* < \bar{\theta}$ ? (B) How do the allocations change with  $N$ ? In other words, what are the signs of  $d\theta^* / dN$  and  $d\theta^f / dN$ ?; and (C) what conditions are needed to have  $Q_p / L_p > Q_o / L_o$  and hence avoid a "peak reversal"? A firm peak case is needed given our assumption that  $D(Q_o, K) = 0$ .

We answer question (A) through the following proposition (proofs of the propositions are in the appendix):

**Proposition 1:**

- (i) If  $(P_p - P_o)/S < \bar{\theta}(B_p - B_o)$ , then the peak period is used, that is  $\theta^* < \bar{\theta}$ .
- (ii) If  $\underline{\theta}B_o < (c + P_o)/S$ , then some consumers will not fly, that is  $\theta^f > \underline{\theta}$ .
- (iii) If  $\bar{\theta}$  is large enough, then the off-peak period is used, that is  $\theta^* > \theta^f$ .

Part (i) says that the peak period is used if the airport price differential between peak and off-peak is not too large. Specifically, the per-passenger airport price differential has to be smaller than the incremental benefit, for the highest consumer type, of changing from the off-peak to the peak. Clearly, when the airport does not practice PLP, the peak is always used. Part (ii) says that if  $\underline{\theta}$  is low enough, then some consumers will not fly. In particular, the lowest consumer type must have a willingness to pay for off-peak travel that is smaller than the airlines' per-passenger marginal cost for an off-peak flight. Finally, part (iii) implies that Brueckner (2002, 2005)'s *single crossing property*, which imposes that  $B_p(\theta) < B_o(\theta)$  for small  $\theta$  values, is not needed to have a non-empty off-peak. This is desirable because that property appears contradictory with the idea that the peak and the off-peak are vertically differentiated. The proof of part (iii) also reveals that a smaller airport price differential between peak and off-peak increases the likelihood of the off-peak been used.<sup>2</sup>

We now answer question (B) regarding the changes of sub-game equilibrium traffic volumes with respect to  $N$ :

**Proposition 2:**

- (i) In the sub-game equilibrium, the number of passengers in the peak period increases with  $N$ , that is  $d\theta^*/dN < 0$ .
- (ii) In the sub-game equilibrium,  $dQ_p/dN < Q_p/(N(N+1))$ .
- (iii) In the sub-game equilibrium, the total number of passengers traveling increases with  $N$ , that is  $d\theta^f/dN < 0$ .
- (iv) In the sub-game equilibrium, if the off-peak period is used for all  $N$  (proposition 1.3), then the number of passengers using the off-peak period increase with  $N$ .

Note that (ii) shows that the (positive) elasticity of total peak demand with respect to the number of airlines,  $\varepsilon_p \equiv -(dQ_p/dN)(N/Q_p)$  is such that  $\varepsilon_p < 1/(N+1)$ , and becomes smaller and smaller as  $N$  gets larger.

As for the possibility of a peak-reversal, i.e.  $Q_p/L_p < Q_o/L_o$ , it is pretty obvious that this may occur only if two things happen simultaneously; first, a large airport price differential between peak and off-peak; second,  $L_p$  and  $L_o$  being similar. The maintained assumption is that  $L_o \gg L_p$ , which is enough to avoid a peak reversal case and therefore to make the assumption  $D(Q_o, K) = 0$  reasonable. See the appendix for more details.

## FARE AND WELFARE COMPARISONS IN THE OUTPUT MARKET

### *Peak and Off-peak Fares*

The final ingredient to characterize the sub-game equilibrium in the output market has to do with the important issue of peak and off-peak equilibrium air fares: how do they compare? We now show that, if the airport charges uniform prices or if the airport peak price is higher ( $P_p \geq P_o$ ), airfares are, as expected, higher during the peak. From (6) and (7) we get that

$$t_p - t_o = \Delta t_{p-o} = \bar{\theta}(B_p - B_o) - Q_p S(B_p - B_o) - \alpha D(Q_p, K) \quad (13)$$

From the equilibrium condition (12), we obtain an expression for  $\bar{\theta}(B_p - B_o)$  that we replace in (13). Hence, in the sub-game oligopoly equilibrium, the difference between peak and off-peak fares is given by

$$\begin{aligned} \Delta t_{p-o} \Big|_{\substack{\text{oligopoly} \\ \text{sub-game eq}}} &= \frac{(P_p - P_o)}{S} + \frac{\beta}{S} D(Q_p) \\ &+ \frac{\beta}{S} \frac{Q_p}{N} D_Q(Q_p) + \alpha \frac{Q_p}{N} D_Q(Q_p) + Q_p \frac{(B_p - B_o)S}{N} \end{aligned} \quad (14)$$

From here it is clear that if  $P_p \geq P_o$ , then  $\Delta t_{p-o} > 0$ . To further interpret (14), first note that  $d\Delta t_{p-o} / dN$  is negative; just differentiate (13) and recall that equilibrium  $Q_p$  and  $Q_o$  increase with  $N$ . This implies that a monopoly airline would have the largest air fare differential. Since  $dt_o / dN$  is also negative (see equation 6), the lower the  $N$ , the larger the off-peak air fare. These two observations are consistent with what we have already showed in proposition 2. Next, it can be seen that for  $N$  very large, the air fare differential approaches the difference between an airline's peak and off-peak per-passenger average cost (first and second term on the RHS). When there is an oligopoly, however, three extra terms are added. The third term in the RHS is the cost of extra congestion on an airline's own flights and caused by an extra passenger flying in the peak. Thus, the first three terms represent the difference between an airline's peak and off-peak marginal costs. The fourth term represents the money value of extra congestion to an airline's passengers when a new passenger chooses the peak. And the fifth term is a mark-up that arises from exploitation of market power. Hence, as it is now known, airlines in oligopoly only internalize (charge for) the congestion they impose on their own flights, and which has two components: extra costs for the airline, and extra delays for its passengers (Brueckner, 2002). When there is a monopoly airline, congestion is perfectly internalized but market power is at its ceiling. When  $N$  is large, congestion is imperfectly internalized but the market power mark-up is small.

### *Social Welfare Comparison*

We now look at the case when a social planner maximizes total surplus in the sub-game. This will be useful to better understand airport's pricing later. We first need a measure of consumer surplus ( $CS$ ). Given the linearity of the conditional indirect utility function,  $CS$  is given by:

$$CS = \int_{\theta^*}^{\bar{\theta}} [\theta B_p - \alpha D(Q_p, K) - t_p(Q_p, Q_o)] f(\theta) d\theta + \int_{\theta^f}^{\theta^*} [\theta B_o - t_o(Q_p, Q_o)] f(\theta) d\theta \quad (15)$$

Using (6) and (7) for  $t_o$  and  $t_p$ , solving the integrals, replacing  $\theta^*$  and  $\theta^f$  with (5) and rearranging, we finally get:

$$CS = \frac{S^2}{2} (B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2) \quad (16)$$

We are then interested in maximizing  $CS + \sum_{i=1}^N \phi^i$ . First-order conditions and imposition of symmetry lead to two equations, analogous to (10) and (11), which characterize the optimum. Subtracting them we get

$$Q_p (B_p - B_o) S^2 + (\alpha S + \beta) (D(Q_p) + Q_p D_Q(Q_p)) + (P_p - P_o) - \bar{\theta} S (B_p - B_o) = 0 \quad (17)$$

With this condition, we can find what the efficient difference between peak and off-peak air fares is. Using (17) to obtain a new expression for  $\bar{\theta} (B_p - B_o)$  to replace in (13), we get:

$$\Delta t_{p-o} \Big|_{\text{sub-game eq}}^{\text{optimal}} = \frac{(P_p - P_o)}{S} + \frac{\beta}{S} D(Q_p) + \frac{(\alpha S + \beta)}{S} Q_p D_Q(Q_p) \quad (18)$$

It is easy to see here that the optimal air fare differential is equal to the difference between an airlines peak and off-peak average costs (first and second term on the RHS), plus all the external costs associated to a new flyer in the peak, i.e. the extra congestion cost of all airlines and passengers, not only that of the airline that is carrying the new peak passenger.

We can then calculate the difference between oligopoly and optimal air fare differentials:

$$\left| \Delta t_{p-o} \right|_{\text{sub-game eq}}^{\text{oligopoly}} - \left| \Delta t_{p-o} \right|_{\text{sub-game eq}}^{\text{optimal}} = Q_p \frac{(B_p - B_o) S}{N} - \frac{(N-1)}{N} \frac{(\alpha S + \beta)}{S} Q_p D_Q(Q_p) \quad (19)$$

This is obviously not signed a priori. It will be positive for small values of  $N$  but negative for larger values of  $N$ . (19) implies that a monopoly airline has an air fare differential that is too large, while a more competitive market has a price differential that is too small. Note that, although there may exist an  $N$  that exactly induces the right air fare differential, it would not reproduce the first best because market power distorts the value of  $t_o$  away from the optimum.

### *Sub-game Cartel: Colluding Airlines*

This case will also be useful to better understand the airport pricing later. Here, we are interested in maximizing  $\sum_{i=1}^N \phi^i$ . The first-order conditions and imposition of symmetry lead to two equations, analogous to (10) and (11), which characterize the optimum. Subtracting them we obtain

$$2Q_p(B_p - B_o)S^2 + (\alpha S + \beta)(D(Q_p) + Q_p D_Q(Q_p)) + (P_p - P_o) - \bar{\theta}S(B_p - B_o) = 0 \quad (20)$$

With this condition, we can find the difference between peak and off-peak airfares in this cartel case. Using (20) to obtain a new expression for  $\bar{\theta}(B_p - B_o)$  to replace in (13), we get:

$$\Delta t_{p-o} \Big|_{\substack{\text{cartel} \\ \text{sub-game eq}}} = \frac{(P_p - P_o)}{S} + \frac{\beta}{S} D(Q_p) + \frac{(\alpha S + \beta)}{S} Q_p D_Q(Q_p) + Q_p(B_p - B_o)S \quad (21)$$

Here, the airfare differential is equal to the difference between an airline's peak and off-peak average costs (the first and second terms on the RHS), plus all the external costs associated to a new flyer in the peak. The cartel, then, as in the social welfare case, internalizes the congestion costs of all carriers and passengers. Here however, there is a fourth term which increases the difference. This term has to do with the "business stealing" effect: since oligopoly airlines behave in non-cooperative fashion, they produce too much with respect to the optimum for the airlines as a whole. This is so because they fail to consider the profits lost by the other airlines when they increase output, depressing prices. In the cartel case then, the air differential has to be larger; the cartel, as a monopoly, is interested in having a less used peak. In fact, it is clear that the cartel airfare differential is identical to the monopoly's; see (14) and impose  $N=1$ . Cartel and monopoly's traffic volumes will differ though since cost functions are convex and not flat.

We can then calculate the difference between oligopoly and cartel airfare differentials:

$$\left| \Delta t_{p-o} \right|_{\text{oligopoly}} - \left| \Delta t_{p-o} \right|_{\text{cartel}} = -\frac{(N-1)}{N} \frac{(\alpha S + \beta)}{S} Q_p D_Q(Q_p) - Q_p \frac{S(B_p - B_o)(N-1)}{N} \quad (22)$$

This is always negative –except when  $N=1$ – and its derivative with respect to  $N$  is negative as well, implying that the difference increases with  $N$ . This makes sense: the price differential of the oligopoly is insufficiently large from the cartel's point of view, and this problem worsens the looser the oligopoly is (i.e. the larger the  $N$ ).

## AIRPORT CHARGE COMPARISONS

To be able to perform these comparisons, we need a good idea about the shapes of the airport's demands. Comparative statics on (10) and (12) help us with this. For example, we can obtain:

$$\begin{aligned} \frac{\partial Q_p}{\partial P_p} &= -\frac{\frac{\partial(-\Omega^p + \Omega^o)}{\partial P_p}}{\frac{\partial(-\Omega^p + \Omega^o)}{\partial Q_p}} \\ &= -\frac{N}{(B_p - B_o)S^2(N+1) + (\alpha S + \beta)((N+1)D_Q(Q_p, K) + Q_p D_{QQ}(Q_p, K))} < 0 \end{aligned} \quad (23)$$

Similarly, we can obtain

$$\begin{aligned}
\frac{\partial Q_p}{\partial P_p} < 0, & \quad \frac{\partial Q_p}{\partial P_o} = -\frac{\partial Q_p}{\partial P_p} > 0, & \quad \frac{\partial Q_p}{\partial K} > 0 \\
\frac{\partial Q_o}{\partial P_p} = -\frac{\partial Q_p}{\partial P_p} = \frac{\partial Q_p}{\partial P_o} > 0, & \quad \frac{\partial Q_o}{\partial P_o} = -\frac{\partial Q_p}{\partial P_o} - \frac{N}{B_o S^2 (N+1)} < 0, & \quad \frac{\partial Q_o}{\partial K} = -\frac{\partial Q_p}{\partial K} \\
\frac{\partial(Q_o + Q_p)}{\partial P_p} = 0, & \quad \frac{\partial(Q_o + Q_p)}{\partial P_o} = -\frac{N}{B_o S^2 (N+1)} < 0, & \quad \frac{\partial(Q_o + Q_p)}{\partial K} = 0
\end{aligned} \tag{24}$$

We can see that, *ceteris paribus*, the airport peak price does not influence total traffic but only the allocation to peak and off-peak periods. The off-peak price then determines the total amount of traffic, while the difference between  $P_p$  and  $P_o$  determines the partition of that traffic into the two periods. Capacity does not influence total traffic either, but only the allocation to peak and off-peak periods. It does this just in the opposite way as the peak-price, that is, larger capacity increases peak traffic.

We have shown that the airport decisions, namely,  $P$  and  $K$ , can influence subsequent output competition. When making its decisions, therefore, the airport will take the second-stage equilibrium outputs into account. These decisions may in reality be set by a private, profit-maximizing airport operator or a welfare-maximizing regulatory authority. Consequently, the objective of an airport may be to maximize profit, or to maximize social welfare. In what follows, we compare airport pricing and capacity investment for these two airport types. In addition, we consider an airport that has some sort of strategic agreement with the airlines using it. For the sake of simplicity of exposition, we consider here that capacity is fixed. Results with variable capacity are available for a congestion pricing model (see Basso, 2005).

### *Private Airport*

Recall that from  $\Omega^o = 0$  in (10) and  $-\Omega^p + \Omega^o = 0$  in (12), we implicitly obtained airport's demands for the peak and off-peak periods,  $Q_o(P_o, P_p, K, N)$  and  $Q_p(P_o, P_p, K, N)$ . A private airport will then maximize profits,  $\pi(P_o, P_p, K, N) = P_o Q_o + P_p Q_p - C(Q_o + Q_p) - rK$ , by choosing  $P_p$ ,  $P_o$  and  $K$ . Note that we assumed, as it is usual in the literature, that operational and capital costs are separable. Also, we assumed constant operational marginal cost for the airport, as evidence show that economies of scale are exhausted at fairly low levels of traffic (Doganis, 1992). First-order conditions lead to:

$$P_o^\pi - C = \frac{P_o^\pi}{\varepsilon_{oo}} + \frac{(P_p^\pi - C)Q_p \varepsilon_{po}}{Q_o \varepsilon_{oo}} \tag{25}$$

$$P_p^\pi - C = \frac{P_p^\pi}{\varepsilon_{pp}} + \frac{(P_o^\pi - C)Q_o \varepsilon_{op}}{Q_p \varepsilon_{pp}} \tag{26}$$

where  $\varepsilon_{oo} = -\frac{\partial Q_o}{\partial P_o} \frac{P_o}{Q_o}$  is the own price elasticity,  $\varepsilon_{po} = \frac{\partial Q_p}{\partial P_o} \frac{P_o}{Q_p}$  is a cross-price elasticity, and  $\varepsilon_{pp}$  and  $\varepsilon_{op}$  are defined analogously. Since  $\partial Q_p / \partial P_o > 0$  and  $\partial Q_o / \partial P_p > 0$  –see (24)– both  $\varepsilon_{op}$  and  $\varepsilon_{po}$  are positive, implying that prices are higher than if peak and off-peak charges were chosen independently. If that was the case, mark-ups over marginal costs would be the usual, i.e. proportional to the inverse of own price elasticity. This is a well-known result for the case of multiproduct monopolies that produce substitutes.

We can simplify the pricing equations and show that  $P_p^\pi > P_o^\pi$ . To do this, replace the elasticities' definitions and simplify, use the fact that  $\partial Q_o / \partial P_p = -\partial Q_p / \partial P_o$  and then use equation (23). We get

$$P_o^\pi = C + \frac{Q_o S^2 B_o (N+1)}{N} + \frac{Q_p S^2 B_o (N+1)}{N} \quad (27)$$

$$P_p^\pi - P_o^\pi = \frac{(\alpha S + \beta)}{N} Q_p \left[ (N+1) D_Q(Q_p, K) + Q_p D_{QQ}(Q_p, K) \right] + \frac{Q_p (B_p - B_o) S^2 (N+1)}{N} \quad (28)$$

Hence, in effect, the private airport charges more during the peak period, and this is true for any  $N$ : as we argued in the introduction, the private airport has an incentive to use peak-load pricing. Note also that the off-peak price, which determines total traffic, is above marginal cost as a result of monopoly power from the part of the airport. There is a double marginalization problem then, which is typical of uncoordinated vertical structures.

Lastly,  $d\pi/dN = \sum \pi_h P_h^\pi + \pi_K K_N^\pi + \pi_N = \pi_N = \sum P_h^\pi \cdot \partial Q_h / \partial N > 0$ , i.e., the private airport prefers a large  $N$ .

### *Public Airport*

The public airport chooses  $P_p, P_o$  and  $K$  to maximize a social welfare function, which is given by  $SW(P_o, P_p, K, N) = P_o Q_o + P_p Q_p - C(Q_o + Q_p) - rK + CS + \Phi$ , where  $CS$  is consumer surplus (see equations 16 and 17), and  $\Phi$  represents sub-game equilibrium total profits for the airline industry as a whole in oligopoly. Since the downstream equilibrium is symmetric,  $\phi^i(Q_h^i, Q_h^{-i}, P_h, K) = \phi^1(Q_o(P_h, K, N), Q_p(P_h, K, N), P_h, K)$  are airlines' equilibrium profits.

We can then easily calculate  $\Phi$  as  $\Phi(P_h, K, N) = N \cdot \phi^1(P_h, K, N)$ , that is

$$\begin{aligned} \Phi(P_h, K, N) = & \bar{\theta} S (B_p Q_p + B_o Q_o) - S^2 (B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2) - (\alpha S + \beta) Q_p D(Q_p) \\ & - Q_o (c + P_o) - Q_p (c + P_p) \end{aligned} \quad (29)$$

We do not include a budget constraint in the public airport problem, noting that fixed fees would solve the problem of budget adequacy. If lump sum transfers are not feasible, then Ramsey-Boiteaux prices should be considered (see Basso, 2005 for more discussion on this).

Replacing  $CS$  from (17) and  $\Phi$  from (28) in the social welfare function, we obtain

$$SW = \bar{\theta}S(B_p Q_p + B_o Q_o) - c(Q_p + Q_o) - C(Q_p + Q_o) - Kr - S^2(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2)/2 - (\alpha S + \beta)Q_p D(Q_p) \quad (30)$$

Derivation of pricing formulas follows from first-order conditions. Using equations (10), (12) and (24) we get:

$$P_o^w = C - \frac{Q_o S^2 B_o}{N} - \frac{Q_p S^2 B_o}{N} \quad (31)$$

$$P_p^w - P_o^w = \frac{(N-1)}{N}(\alpha S + \beta)Q_p D_Q(Q_p, K) - \frac{Q_p S^2 (B_p - B_o)}{N} \quad (32)$$

The interpretation of the public airport's pricing rules is as follows:

- The public airport pricing can be seen as decided in two phases; first, it induces the right amount of total traffic by choosing a  $P_o^w$  below the airport's marginal cost. This is needed because, in the airline market, market power induces allocative inefficiencies. The public airport fixes this inefficiency by subsidizing the airlines and hence lowering their marginal costs in the off-peak. The exact amount of the subsidy depends on the extent of the market power, which is why it depends on  $N$ .
- In the second phase, and once the total traffic is set to its optimal level, the public airport chooses exactly the price differential that will induce the optimal air fare differential downstream; this is apparent from equations (19) and (32). In this way, the airport induces the optimal allocation to peak and off peak periods. In sum, the airport manages to obtain a first-best outcome.
- As explained for equation (19), the price differential is not signed a priori; hence, *it may happen that the airport charge is smaller in the peak!* The price differential will be negative for small  $N$ . This is so because a tight airline oligopoly has an air fare differential that is too large due to extreme market power, while congestion is reasonably internalized; the airport price differential then is driven by the market power effect (second term). When  $N$  is large, the airport price differential will be positive. This is so because a loose oligopoly would have an air fare differential that is too small due to uninternalized congestion, while market power is weak; the airport price differential then is driven by the congestion effect (first term).
- Note that, even if the airport peak price is below the off-peak price, the air fare differential downstream will always be positive because the airport price differential is calculated to exactly generate the socially optimal air fare differential, which as discussed in (18), will be given by  $\beta D(Q_p)/S + (\alpha S + \beta)Q_p D_Q(Q_p)/S > 0$ .
- Brueckner (2002) identified the first term in (36) as the toll per flight that should be charged by the airport authorities to address the problem of uninternalized congestion. Pels and Verhoef (2004) and Basso (2005) pointed out that the optimal toll should also include the second term, the market power effect<sup>3</sup>; they did this however, using models of congestion pricing (one period) and not peak load-pricing, as here and in Brueckner (2002). This is important because a toll equal to the two terms, congestion and market power effects, will not be optimal unless charged on top of the optimal charge in the off-

peak, *which is not marginal cost*. In other words, only analyzing the toll that should be charged during peak hours is only a partial view of the problem

- Note that if lump sum transfers (two-part tariffs) are unfeasible, the pricing rules previously discussed may lead to airport's budget inadequacy. If budget adequacy has to be ensured but lump sums are not feasible, then the first best may not be attainable: marginal prices would have to do both, align incentives and transfer surplus, making the airport fall short of "control instruments" (Mathewson and Winter, 1984).

In this model, the public airport is indifferent between values of  $N$  since  $dSW/dN = \partial SW/\partial N = 0$ . Basso (2005) showed that if airlines are not homogenous or if passengers were affected by schedule delay cost, this would not be the case. Also, if budget adequacy is a problem, a larger  $N$  may be preferred as  $P_o^W$  gets closer to marginal cost, and the peak price exceeds the off-peak price.

#### *Airport-Airline Joint Profit Maximization*

The reasons why it is interesting to look at this case are two-fold: on one hand, a simple pricing mechanism, two-part tariff, is enough for the joint maximization of profits outcome to arise. On the other hand, it has been often argued that more strategic collaboration between airlines and airports may make price regulation unnecessary. The analysis of joint maximization of profits then works as a benchmark case. See Basso (2005) for a more detailed discussions on this issue.

The problem faced by this airport is:  $\max_{P_o, P_p, K} \pi + \Phi$ . Using  $\Phi$  in (28), this can be re-written as:

$$\begin{aligned} \max_{P_o, P_p, K} \pi + \Phi = & \bar{\theta}S(B_p Q_p + B_o Q_o) - c(Q_p + Q_o) - C(Q_p + Q_o) - Kr \\ & - S^2(B_o Q_o^2 + 2B_o Q_o Q_p + B_p Q_p^2) - (\alpha S + \beta)Q_p D(Q_p) \end{aligned}$$

Derivation of pricing formulas follows from first-order conditions. Using equations (10), (12) and (24) we get:

$$P_o^{JP} = C + \frac{Q_o S^2 B_o (N-1)}{N} + \frac{Q_p S^2 B_o (N-1)}{N} \quad (33)$$

$$P_p^{JP} - P_o^{JP} = \frac{(N-1)}{N} (\alpha S + \beta) Q_p D_Q(Q_p, K) + \frac{Q_p S^2 (B_p - B_o)(N-1)}{N} \quad (34)$$

The interpretation of the joint profits airport's pricing rules is as follows:

- As before, this airport can be seen as deciding its prices in two phases; first, it induces a contraction of total traffic by choosing a  $P_o^{JP}$  above marginal cost. It does this because in the airline market, failure of coordination among the airlines induces them to produce too much with respect to what is best for them as a whole. The amount of excess production depends on how tight the oligopoly is, which is why the off-peak mark-up decreases with  $N$ . In particular, when the airline market is monopolized, the airport does not need the mark-up at all.

- In the second phase, the airport chooses exactly the (non negative) price differential that will induce the cartel air fare differential downstream; this is apparent from equations (21) and (34). In this way, the airport induces the allocation to peak and off- peak periods that maximizes airlines profits.
- In sum, the airport manages to obtain the cartel outcome, destroying airline competition. This result, which was obtained by Basso (2005) in a congestion pricing setting (without peak-load pricing), has different intuitions depending on why the maximization of joint profits was the relevant case. With two-part tariffs, the private airports use the variable prices, peak and off-peak, to destroy competition downstream in order to maximize the profits of airlines, which are later captured through the fixed fee. When the max joint profits case arises because of collaboration between airlines and airports, what happens is that airlines would like to collude in order to increase profits, but fail to do so because of the incentives to defect on any possible agreement. What they manage to do here, however, is to ‘capture’ an input provider to run the cartel for them. By altering the price of the input, and therefore the downstream marginal costs, in both the peak and the off-peak periods, the input provider induces both the collusion level of total output and the right allocation of consumers to the peak and the off-peak. The upstream firm is rewarded with part of the profits, which is where bargaining power enters the picture.
- The airport pricing rules take into account both, the congestion externality and the business-stealing effect at both periods: the airport’s price differential has two parts, as in equation (21).<sup>4</sup> Note that when  $N=1$ , there is no business-stealing effect and congestion is perfectly internalized by the monopolist; consequently, the two terms vanish: *the airport will not use peak-load pricing!*
- Despite the fact that the result is as if airlines collude, this is not necessarily worse for social welfare than a private airport charging linear prices as before because, here, two other harmful externalities are dealt with, the vertical double marginalization and the congestion externality.

### *Comparisons of Prices for Given Capacity*

In this subsection, we compare the prices that each type of airport would set. By looking at equations (27) and (28) and (31) to (34), and defining  $\Delta P_{p-o} \equiv P_p - P_o$  we can enounce the following proposition (the proof is direct from the pricing rules).

**Proposition 3:** For a fixed capacity, the airport prices fulfill

- (i)  $P_o^W < P_o^{JP} < P_o^\pi$
- (ii)  $\Delta P_{p-o}^W < \Delta P_{p-o}^{JP} < \Delta P_{p-o}^\pi$ .

What part (i) shows is that the public airport will induce the largest amount of total traffic, whilst the private airport will induce the smallest. That is, a private airport induces allocative inefficiencies. Strategic collaboration between airlines and the airport smoothes the problem, but recall that downstream airfares will be as if airlines collude. Additionally, part (ii) shows that the relative allocation of passengers to peak and off-peak periods are different. Since the public airport has the smallest price differential, it will have the largest ratio of peak to off-peak traffic. Conversely, a private airport will have the smallest ratio. With a private airport, the peak would be underused not only because the airport contracts total traffic but also

because its price differential is too large. Again, airports with strategic agreements with airlines represent a middle-of-the-road case.

## CONCLUDING REMARKS

In this paper, we have analyzed the sequential peak-load pricing problem that arises because airports are input providers for a final market that faces a periodic demand. We have analyzed this PLP problem for a private unregulated airport, for a public airport maximizing social welfare, and for an airport that strategically collaborates with the airlines and hence maximizes joint profits. We found that, for a fixed capacity, privatization would not induce efficient peak-load pricing structures as it has been argued in the literature. While a private airport always has an incentive to use PLP –higher airport charge in the peak–, even when the airlines use PLP and irrespective of the number of airlines servicing the airport, the pricing structure the private airport chooses would induce insufficient total traffic and a ratio of peak to off-peak traffic that is too small. Somewhat surprisingly, depending on the degree of market power –that is, the number of firms at the airport–, a public airport may find it optimal to have a peak price that is lower than the off-peak price. An airport that strategically collaborates with airlines would induce greater total traffic and have a larger peak to off-peak traffic ratio than a pure private airport, but both numbers will still be smaller than those for a public airport. If the airport collaborates with a dominant airline, it would not use peak-load pricing.

Finally, we note that although the airline industry is chosen for analysis, our basic model structure, in which airports make their pricing and capacity decisions prior to airlines' strategic interactions in the final product market, is highly relevant to many other industries, such as electricity, telecommunications, and transport terminals (e.g., sea ports-carriers-shippers). In telecommunications, for example, at the upstream level there are the network owners and downstream there are carriers who must use the network to produce the final good which is telephone calls. Furthermore, like airports, these industries (electricity, ports and telecommunications) are undergoing privatization and corporatization in a number of countries. Our results would therefore contribute not only to a better understanding of airport policy and management, but also to the advancement of the theory and methodology for analyzing peak-load pricing in a general setting.

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## APPENDIX

- *Proof of proposition 1*

First, equivalent conditions for interior allocations, but in terms of  $Q_p$  and  $Q_o$  are:

The peak is used:  $\theta^* < \bar{\theta} \Leftrightarrow (\bar{\theta} - \theta^*)/S > 0 \Leftrightarrow Q_o > 0$

Some consumers do not fly:  $\theta^f > \underline{\theta} \Leftrightarrow (\bar{\theta} - \theta^f)/S < (\bar{\theta} - \underline{\theta})/S \Leftrightarrow Q_o + Q_p < (\bar{\theta} - \underline{\theta})/S$

The off-peak is used:  $\theta^* > \theta^f \Leftrightarrow (\theta^* - \theta^f)/S > 0 \Leftrightarrow Q_o > 0$

With this, the proofs of each part are:

(i) Note that  $(-\Omega^p + \Omega^o)$  in (12) is strictly increasing in  $Q_p$ , and  $-\Omega^p + \Omega^o|_{Q_p \rightarrow \infty} > 0$ . Also,

$$-\Omega^p + \Omega^o|_{Q_p=0} = (P_p - P_o) - \bar{\theta}S(B_p - B_o) . \text{ Hence, if } P_p - P_o < \bar{\theta}S(B_p - B_o) , \text{ then}$$

$$-\Omega^p + \Omega^o|_{Q_p=0} < 0 \text{ and } Q_p > 0 . \quad \blacksquare$$

(ii) From  $\Omega^o=0$  in (10) we get that  $(Q_o + Q_p)B_oS^2(N+1)/N = B_o\bar{\theta}S - c - P_o$ , This imply that

$$Q_o + Q_p < (B_o\bar{\theta}S - c - P_o)/(B_oS^2) . \text{ Hence, a sufficient condition for } Q_o + Q_p < \frac{\bar{\theta} - \underline{\theta}}{S} \text{ is:}$$

$$(B_o\bar{\theta}S - c - P_o)/(B_oS^2) < \bar{\theta} - \underline{\theta}/S , \text{ which leads to } \underline{\theta}B_o < (c + P_o)/S . \quad \blacksquare$$

(iii) From  $\Omega^o=0$  we know that  $Q_o + Q_p = \frac{(B_o\bar{\theta}S - c - P_o)N}{B_oS^2(N+1)}$ . Hence,  $Q_o > 0$  is equivalent to

$$Q_p < \frac{(B_o\bar{\theta}S - c - P_o)N}{B_oS^2(N+1)} \equiv \tilde{Q}_p . \text{ In order to ensure } Q_p < \tilde{Q}_p , \text{ we need that}$$

$$-\Omega^p + \Omega^o|_{Q_p=\tilde{Q}_p} > 0 \text{ (see proof of part i). Straightforward algebra gives us}$$

$$(-\Omega^p + \Omega^o)(\tilde{Q}_p) = (\alpha S + \beta) \left[ D(\tilde{Q}_p) + \frac{\tilde{Q}_p}{N} D_Q(\tilde{Q}_p) + \frac{(P_p - P_o)}{(\alpha S + \beta)} - \frac{(B_p - B_o)(c + P_o)}{B_o(\alpha S + \beta)} \right], \text{ so}$$

that a sufficient condition for  $|\Omega^p + \Omega^o|_{\tilde{Q}_p} > 0$  is

$$D(\tilde{Q}_p) > \frac{(B_p - B_o)(c + P_o)}{B_o(\alpha S + \beta)} - \frac{(P_p - P_o)}{(\alpha S + \beta)}. \text{ And since } \partial \tilde{Q}_p / \partial \bar{\theta} > 0, \text{ the condition is always fulfilled for } \bar{\theta} \text{ large enough.} \quad \blacksquare$$

For a linear delay function  $D(Q_p, K) = \delta Q_p / K$ , the lower bound on  $\bar{\theta}$  can be found explicitly; it is given by  $\bar{\theta} = \frac{SK(N+1)}{\delta B_o(\alpha S + \beta)N} \left( (B_p - B_o)(c + P_o) - B_o(P_p - P_o) \right) + c + P_o$ .

And a lower bound without  $N$ , would be  $2\tilde{\theta}N/(N+1)$ .

- *Proof of proposition 2*

(i) Differentiating  $(-\Omega^p + \Omega^o)$  with respect to  $N$  we get:  $\frac{dQ_p}{dN} = -\frac{\partial(-\Omega^p + \Omega^o)/\partial N}{\partial(-\Omega^p + \Omega^o)/\partial Q_p}$ . But

$$\frac{\partial(-\Omega^p + \Omega^o)}{\partial N} = -\frac{Q_p S^2 (B_p - B_o)}{N^2} - \frac{(\alpha S + \beta) Q_p D_Q(Q_p)}{N^2} < 0, \text{ and}$$

$$\frac{\partial(-\Omega^p + \Omega^o)}{\partial Q_p} = \frac{S^2(N+1)(B_p - B_o)}{N} + (\alpha S + \beta) \left( \frac{N+1}{N} D_Q(Q_p) + \frac{Q_p}{N} D_{QQ}(Q_p) \right) > 0, \text{ ,}$$

so that  $\frac{dQ_p}{dN} > 0$  and therefore  $d\theta^* / dN < 0$  from equation (5). ■

(ii) From the proof of part (i), we get

$$\frac{dQ_p}{dN} = \frac{\frac{Q_p S^2 (B_p - B_o)}{N^2} + \frac{(\alpha S + \beta) Q_p D_Q(Q_p)}{N^2}}{\frac{S^2(N+1)(B_p - B_o)}{N} + (\alpha S + \beta) \left( \frac{N+1}{N} D_Q(Q_p) + \frac{Q_p}{N} D_{QQ}(Q_p) \right)}, \text{ which can be}$$

written as

$$\frac{dQ_p}{dN} = \frac{Q_p}{N(N+1)} \left( \frac{S^2(B_p - B_o) + (\alpha S + \beta) D_Q(Q_p)}{S^2(B_p - B_o) + (\alpha S + \beta) D_Q(Q_p) + \frac{(\alpha S + \beta) Q_p D_Q(Q_p)}{N+1}} \right), \text{ from which}$$

the result follows. ■

(iii) From  $\Omega^o=0$  we know that  $Q_o + Q_p = \frac{(B_o \bar{\theta} S - c - P_o)N}{B_o S^2 (N+1)}$ , from where it is direct that

$$\frac{d(Q_o + Q_p)}{dN} = \frac{(B_o \bar{\theta} S - c - P_o)}{B_o S^2 (N+1)^2} > 0 \text{ and therefore } d\theta^f / dN < 0 \text{ from equation (5).} \quad \blacksquare$$

(iv) From the proof of part (iii) we get  $\frac{dQ_o}{dN} = \frac{(B_o\bar{\theta}S - c - P_o)}{B_oS^2(N+1)^2} - \frac{dQ_p}{dN}$ . From proposition 2.2,

we further obtain  $\frac{dQ_o}{dN} > \frac{(B_o\bar{\theta}S - c - P_o)}{B_oS^2(N+1)^2} - \frac{Q_p}{N(N+1)}$ . If the off-peak is used for all  $N$ ,

then  $Q_p < \frac{(B_o\bar{\theta}S - c - P_o)N}{B_oS^2(N+1)} \equiv \tilde{Q}_p$  (see the proof of proposition 1.3). Hence

$\frac{dQ_o}{dN} > \frac{(B_o\bar{\theta}S - c - P_o)}{B_oS^2(N+1)^2} - \frac{(B_o\bar{\theta}S - c - P_o)N}{B_oS^2(N+1)} \frac{1}{N(N+1)}$  and, therefore  $dQ_o/dN > 0$ . ■

- $L_o \gg L_p$  precludes a peak-reversal

The no peak reversal condition can be written as  $Q_p L_o / L_p > Q_o$ . Since we know from  $\Omega^o=0$  that  $Q_o + Q_p = (B_o\bar{\theta}S - c - P_o)N / (B_oS^2(N+1))$ , the no peak-reversal condition becomes

$Q_p > \frac{(B_o\bar{\theta}S - c - P_o)N}{B_oS^2(N+1)} \frac{L_p}{L_p + L_o}$ , and hence a sufficient condition is

$Q_p > \frac{(B_o\bar{\theta}S - c - P_o)}{B_oS^2} \frac{L_p}{L_p + L_o}$ , which always hold if  $L_o \gg L_p$ .

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## ENDNOTES

<sup>1</sup> *Proof:* (i) if  $\theta_1$  flies,  $\theta_1 B_h - \alpha D_h - t_h \geq 0$  for  $h=p,o$ . If  $\theta \geq \theta_1$ ,  $\theta B_h - \alpha D_h - t_h \geq \theta_1 B_h - \alpha D_h - t_h \geq 0$  and  $\theta$  flies. (ii) is analogous. (iii) consider  $h(\theta) = \theta(B_p - B_o) - \alpha(D_p - D_o) - (t_p - t_o)$  and suppose  $\theta$  flies. Then if  $h(\theta) \geq 0$ ,  $\theta$  chooses to fly in the peak. If  $h(\theta) < 0$ ,  $\theta$  chooses to fly in the off-peak. Now, suppose  $\exists \theta^*$  such that  $h(\theta^*) = 0$  (interior solution). Then, since  $h'(\theta) > 0$ , if  $\theta \geq \theta^*$ ,  $\theta$  chooses the peak and if  $\theta < \theta^*$ ,  $\theta$  chooses the off-peak ■

<sup>2</sup> The lower bound for  $\bar{\theta}$  cannot be made explicit because of the non-linearity of the delay function but can be expressed in closed form if a linear delay function is used. See the appendix.

<sup>3</sup> To be fair, although Brueckner did not formally consider the second term in the toll to be charged, he did point out that, depending on the size of the market power term, a pure congestion toll could be detrimental for social welfare.

<sup>4</sup> This idea of an upstream firm running the cartel for the downstream firms has been discussed in the vertical control literature and, particularly, in the input joint-venture case. For example, Chen and Ross (2003) formalized the conjecture that input joint-ventures can facilitate collusion and push a market toward the monopoly outcome. If airport provision was seen as an input joint-venture by the airlines, our results show three things in addition to what Chen and Ross found. First, that the results hold even in a peak-load pricing setting, i.e. when demand is periodic. Second, that if there are externalities, the input prices are, additionally, used to force their internalization by downstream competitors. And third, that when marginal costs downstream are not constant, the outcome is not as in monopoly or a downstream merger, but as in a cartel.