

# **Improving Traffic Assignment Model Using Intersection Delay Function**

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## ABSTRACT

In this paper a method is proposed to enhance traffic assignment process in traditional travel demand forecasting models. In this method link delay is calculated as a function of flow on that link as well as flow on adjacent links, using intersection delay calculations. This method employs a combination of Frank-Wolf and the method of successive averages to model multi-path vehicle assignment in a reasonable amount of computational time, for small and medium size transportation networks.

Inclusion of volume-based intersection delays in regional planning models is not yet widespread in practice. The objective of this paper is to consider the impact of volume-based (dynamic) intersection delays on multi-path traffic assignments in medium size transportation networks. The approach selected is designed to provide more realistic results than those generated by models represents only link delays, due to considering flows on other links. Convergence has been an issue in assignment models that use intersection delay. Nevertheless, convergence has been observed in a reasonable amount of time in our model.

The results of this paper suggest that intersection delay, as especially at unsignalized intersections where delays at minor legs are highly dependent on volumes, result in significantly different assignments. It has been also demonstrated that the model achieves a higher levels of calibration and it is more sensitive to intersection level policies.

## INTRODUCTION

The most popular travel demand forecasting models that are extensively used by transportation modelers for decades are known as four-step models. Traffic assignment is a key element in travel demand forecasting process that assigns travel demands into transportation supply (network) based on cost of travel. The travel cost is usually considered to be travel time between the origin and the destination of travel. The traffic assignment model is used to predict network flows that are associated with future planning scenarios and estimates the link travel times and related attributes that are utilized to estimate benefit analysis and air quality impact.

In order to calculate travel time between origin and destination, a function presenting the relationship between link delays and link flows is used by transportation modelers. This function, which is called Link Performance Function (LPF), is assumed to be positive, increasing, and convex. An unrealistic and restrictive assumption of these functions is that the travel time on each link is independent of flows on other links in the network. On the other hand, intersection delay depends not only on the physical characteristics and control policies of intersections, but also on the traffic flow on other links. Therefore, use of reliable intersection delay along with link delay could yield a more realistic delay for each path between origin and destination

Many four-step models are now using a multi-path vehicle assignment algorithm with only link delays. These models attempt to incorporate intersection delay into approaching link delay. This is often performed simply because adding dynamic turn penalty delays would result in onerous model run times.

The objective of this paper is to enhance traffic assignment process in four-step models by considering the impact of dynamic turn penalty delays on link delays. Since calculating dynamic turn penalty delays (intersection delays) is time consuming, the proposed method is suitable for small or medium size transportation networks. The proposed method is designed to provide more realistic results than those generated by models representing only link delays. On the other hand, the method is designed to be convergent and to reach equilibrium in a reasonable amount of time.

This paper is organized as follows. Section 2 presents a background about intersection delay; Section 3 explains the proposed traffic assignment model. The empirical results in our case study are presented in Section 4, and finally Section 5 summarizes and concludes the paper.

## **BACKGROUND**

As stated earlier, most of current four-step models use multi-path traffic assignment algorithms. In order to guarantee convergence of the algorithms, link delays are considered to be a function of link flows only on that link, which is not realistic. Adding intersection delays to the link delays could increase the preciseness of the delay calculations. However, the model run time increases when calculating intersection delay.

There are two different intersection delays: static and dynamic. The static intersection delay which is supported by many transportation modeling packages, represents the predicted delays at each approach of intersection which is not dependent to flows at intersections. The dynamic intersection delay, computes the volume-based delays at intersections. The problem associated with the dynamic intersection delay is to find a convergent algorithm to find user equilibrium on the network. To the best knowledge of authors, incremental assignment has been used when dynamic intersection delay is calculated.

Frank-Wolf (F-W) decomposition (Sheffi, 1985) has been extensively used by transportation modelers to compute a user equilibrium solution. The only method of equilibrium traffic assignment known to be able to handle difficult delay relationships is method of successive averages (MSA) (Sheffi, 1985). This method produces identical results to F-W algorithm on networks with simple LPF, however convergence is slower (Sheffi and Powell, 1982).

Many transportation modeling packages support the modeling of volume-based intersection delays to some degree, but not all combine this feature with a user equilibrium assignment. TModel and ITM allow fixed and volume-based intersection delays with incremental assignment (TModel, 1992). QRS II allows the inclusion of volume-dependant intersection delays with MSA assignment, but not Frank-Wolfe decomposition (Quick Response System, 2000). Volume-dependant intersection delays have also been implemented with multi-path user equilibrium assignment in EMME 2 (EMME2, 2005), and Cube Voyager allowing volume-dependent intersection delays with multi-path Frank-Wolfe decomposition.

Our intersection delay method is implemented in the TransCAD software package. Intersection delay can be modeled in three different ways in TransCAD: static intersection delay, link performance function, and dynamic intersection delay. In the static intersection delay, users can insert a fixed delay for each approach of all

intersections or intersections by link type. In the link performance function approach, a Logit-based function calibrated by the Israel Institute of Transportation Planning and Research is utilized. The function includes both link delay and intersection delay, but the intersection delay is not volume-based (TransCAD, 2000). A valid method for including volume-dependant intersection delays is not included in TransCAD.

### THE PROPOSED MODEL

We propose a two-stage iterative algorithm to compute user equilibrium when taking into account dynamic intersection delay along with link delays to assign traffic. Our method uses a combination of F-W and MSA in order to reach an equilibrium in finite number of iterations in a reasonable amount of running time. This model is suitable for small and medium-scale networks. In the first stage we use the F-W to compute user equilibrium for a network using link delays as a function of link volumes and static intersection delays. In the second stage, we calculate intersection delays using flows on the links that are directly connected to that intersection. Then, we add these calculated delays to the corresponding approaches from each link to the other links. These delays are added to the static intersection delay table as well. Then using a method similar to MSA and having updated intersection delay table as input, we repeat the F-W assignment algorithm. This procedure is repeated until convergence criterion is satisfied. Convergence criterion is that link delay is not changing in two consecutive iterations. The MSA is applied to our model as follows. The computed intersection delays are added to the link delays computed by the UE. This link delay is stored in the *NewDelay* field and the link delay computed in the previous iteration is stored in the *PrevDelay* field. Link delay (*LinkDelay*) is calculated to be the convex combination of the *NewDelay* and the *PrevDelay*. In this approach, the weight of *NewDelay* decreases with iteration number in order to achieve convergence. This procedure continues until the convergence criterion is satisfied, defined as when the relative difference between the *LinkDelay* and the *PrevDelay* is less than a small number, say .001. The stepwise algorithm is as follows.

- Step 0.* Initialization: set iteration number,  $n=1$ ; *LinkDelay* = Free Flow Travel Time.
- Step 1.* Update *PrevDelay* = *LinkDelay*.
- Step 2.* Perform User Equilibrium (UE) Assignment using Free Flow Travel Time and Turn Penalty table as inputs. Update the Link Volumes ( $V$ ) and *NewDelay* for each link with the UE output
- Step 3.* Calculate the intersection delays. Update the *NewDelay* and the Turn Penalty Table by adding the associated intersection delay.
- Step 4.* Update the  $LinkDelay = (n-1)/n (PrevDelay) + 1/n (NewDelay)$ .
- Step 5.* Calculate  $Delta = \max \{(LinkDelay - PrevDelay)/LinkDelay\}$
- Step 6.* If  $Delta < .001$ , stop; otherwise  $n = n + 1$  and go to Step 1.

FIGURE 1 presents detailed stepwise algorithm. This algorithm is convergent since it uses MSA and F-W which are both convergent (Sheffi, 1985). In the inner iteration,

using F-W method, we find a user equilibrium solution in several iterations, and in the outer iteration, we find the convex combination between the current link delays and link delays in the previous iterations. This procedure converges after few iterations. Therefore, our procedure terminates in a reasonable amount of running time.

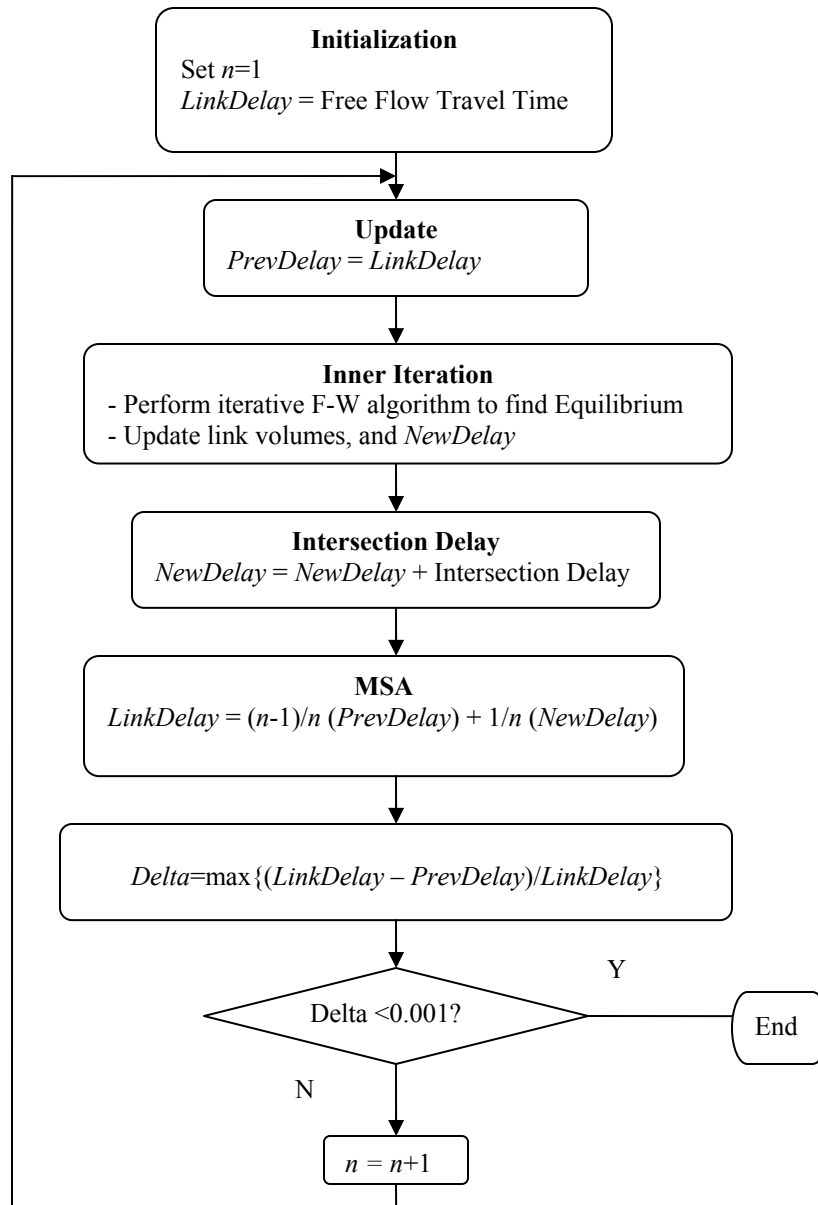
Our method utilizes the intersection delay equation used in the TModel software package, where delay is calculated as

$$Delay_n = \alpha_n \left( \frac{V}{C_n} \right)^{Exp_n} + Const_n$$

where  $V$  is intersection volume and is calculated as

$$V = \sum_{links} V_{approach}$$

and  $C$  is intersection capacity;  $\alpha$ ,  $Exp$ ,  $Const$  are parameters specified for each node  $n$ . The form of this equation was developed via a regression analysis described in Transportation Research Circular 212 (TModel, 1992).



**FIGURE 1 Algorithmic Presentation of our Proposed Method**

## EMPIRICAL RESULTS

We applied three different approaches to assign traffic (our proposed method, static intersection delay, and link delay only) to Chittenden County network in Vermont using TransCAD software package. The Chittenden County Metropolitan Planning Organization (CCMPO) model is primarily a zonal-based model. For the purposes of modeling traffic flow, the completed model includes 335 internal Transportation Analysis

Zones (TAZs) covering the 18 municipalities in Chittenden County. FIGURE 2 presents the network. For modeling purposes, major roadways within the modeling region were selected to represent the entire road network. There are 1,828 road segments represented as links, and 1,353 TAZ's and intersections represented as nodes.

The model is intended to capture an average peak-hour of traffic in the region. To achieve this, the time periods of 7-8 AM and 4-5 PM on a mid-week day (Tuesday, Wednesday, and Thursday) in September was chosen. September was chosen because it is a time during which public schools and colleges are in session, while seasonal (summer) traffic is still observed. 7-8 AM and 4-5 PM were chosen as the AM and PM peak time periods based on September data from automatic traffic recorders throughout the region.

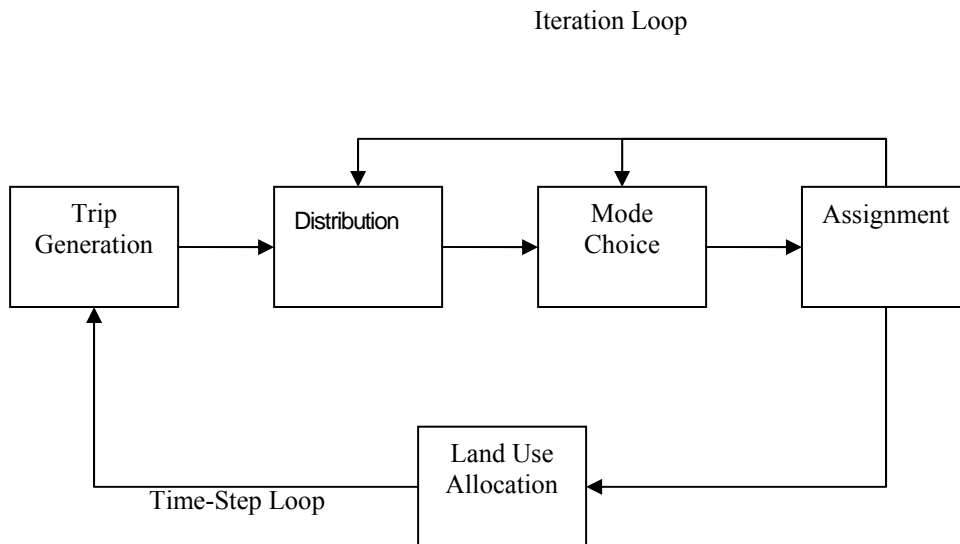
By including a land use allocation module before trip generation, the Model adds a fifth step to the traditional four-step model. The addition of the land use allocation module enables development of consistent and realistic transportation scenarios in transportation planning. It also enables better analyses of transit improvements and trip reduction measures.

An overview of the Model system is shown in FIGURE 3. At the start of a full model run, trip generation uses land use data to calculate trip ends at the transportation analysis zone (TAZ) level. These trip ends are then paired into origins and destinations in the distribution module. In the mode split module, a mode of travel is selected for each trip. Vehicle trips are assigned to the highway network in the assignment module. Finally, the next time period's land use (i.e. housing and jobs) is allocated based in part on accessibility, as measured in part by travel times. The land use increment adds to the existing land use, forming the basis for trip generation for the next time period. This completes one cycle of the model and illustrates how transportation accessibility is the predominant feedback of the model.

As presented in FIGURE 3, there are two feedback loops in the model: time-step loop in which accessibility affects land use allocation, which in turn, works through the various modeling stages, to further affect accessibility; and iteration loop. Accessibility, which is calculated based on outputs from the assignment module, also is an important determinant of trip distribution and mode split. Therefore it is customary to iterate these three modules in order to reach a convergent solution. The iteration loop occurs within a simulation year, because distribution and mode split adjust quickly to changes in service levels. Land use allocation adjusts much more slowly. Therefore, the land use allocation module allocates land use for the next time period based on accessibility in the previous time period. In general, we use time steps of 5 years that imply an average lag of 2 1/2 years. We consider this period to be a reasonable approximation of true time lags present in land use construction decision making.



**FIGURE 2 Chittenden County Network**



**FIGURE 3: Feedback in the Model Structure**

### **Traffic Assignment Using Link Delay**

In this approach we used BPR link performance function (BPR, 1964) to calculate link delays. Link delay is a function of the link volume as follows.

$$T = T_0(1 + \alpha(V/C)^\beta)$$

where,  $T$  is the link delay,  $V/C$  is the ratio of flow on the link to the link capacity, and  $\alpha$  and  $\beta$  are parameters. We calibrated  $\alpha$  and  $\beta$  based on road class for each approach as presented in TABLE 1. In the first approach we calibrated the model in such a way that LPF represents link delays close to the real world delays (delay based on flow in the link and flow of other links), therefore, parameter  $\alpha$  is much higher than the default value in the BPR function. TABLE 2 reports the calibration statistics of the model. The user equilibrium is reached after 59 iterations in this approach.

### **Traffic Assignment Using Link Delay and Static Intersection Delay**

In this approach fixed intersection delay from some links to some other links is added to the model as a table. This table includes turn prohibitions as well. TABLE 2 reports the calibration statistics of the model. The user equilibrium is reached after 37 iterations in this approach.

### **Traffic Assignment Using Link Delay and Dynamic Intersection Delay**

In this approach we implemented our model in which both dynamic intersection delays and link delays are utilized. TABLE 2 reports the calibration statistics of the model. The user equilibrium is reached after 4 iterations (for the outer loop) including 37, 39, 41, and 38 inner iterations, respectively, in each outer iteration. The calibrated  $\alpha$  and  $\beta$  parameters in the BPR function are lower than that in the other two approaches.

Calibration statistics verify that the model is better calibrated using dynamic intersection delay. Comparing other statistics such as vehicle miles traveled (VMT) and vehicle hours traveled (VHT) of the three alternatives shows that we have achieved different assignments and our proposed method yields better results. As presented in TABLE 5, VHT drops considerably when we use intersection delays verifying that using intersection delay gives a better traffic assignment.

For the sake of explaining the three alternatives, we picked two intersections, one signalized and one unsignalized intersection to report link attributes. The signalized intersection named five corners and the unsignalized intersection is Roosevelt Highway and Main Street. These intersections were selected due to having traffic counts on links connected to them. Also, they are important and congested intersections. Figures 4 and 5 present the intersections and tables 4 and 5 report link attributes including link flows, link delays, operational speeds, traffic counts, and free flow travel times.

## **CONCLUSION**

In this paper, we attempt to consider the impact of volume-based (dynamic) intersection delays on multi-path traffic assignments in medium size transportation networks. The proposed approach is designed to calculate volume-based intersection

delay to provide more realistic link flows and link delays than those generated by models represents only link delays, due to considering flows on other links. The proposed method combines the Frank-Wolf decomposition and the method of successive averages to present a convergent and relatively fast method. The proposed method is suitable for small and medium-scale transportation networks due to time consuming intersection delay calculations.

Many researchers and transportation modelers assume that traffic assignment using only link delay gives similar results to than that using intersection delay. Therefore, they ignore time consuming intersection delay calculations. This is true in region level, but volume-based intersection delay leads to different assignment which is more realistic. Therefore, using intersection delay, specially at unsignalized intersections where delays at minor legs are highly dependent on volumes, result in significantly different assignments.

**TABLE 1 Calibrated Parameters for the BPR Function in each Approach**

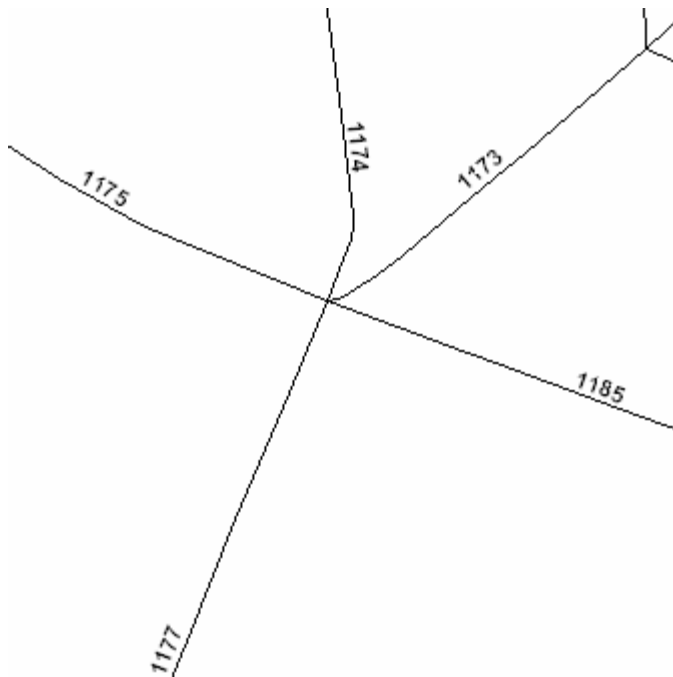
	Class	Link Delay		Static Intersection Delay		Dynamic Intersection Delay	
		$\alpha$	$\beta$	$\alpha$	$\beta$	$\alpha$	$\beta$
<b>Interstate</b>	1	1.65	4	1.65	4	.99	4
<b>Limited Access Highway</b>	2	.33	3.9	.33	3.9	.2	3.9
<b>Principal Arterial</b>	3	1.1	3.9	1.1	3.9	.66	3.9
<b>Minor Arterial</b>	4	.39	3.9	.39	3.9	.23	3.9
<b>Major Collector</b>	5	.28	3.9	.28	3.9	.17	3.9
<b>Urban local</b>	6	6.6	3.9	6.6	3.9	3.96	3.9
<b>Rural Major Collector</b>	7	.75	3.9	.75	3.9	.45	3.9
<b>Ramps</b>	8	.33	3.9	.33	3.9	.2	3.9
<b>Internal Dummy Load Link</b>	9	.66	5.3	.66	5.3	.4	5.3
<b>External Dummy Load Link</b>	10	.66	5.2	.66	5.2	.4	5.2
<b>Coordinated Signal</b>	11	1.65	2	1.65	2	.99	2

**TABLE 2 Calibration Statistics of the Three Approaches**

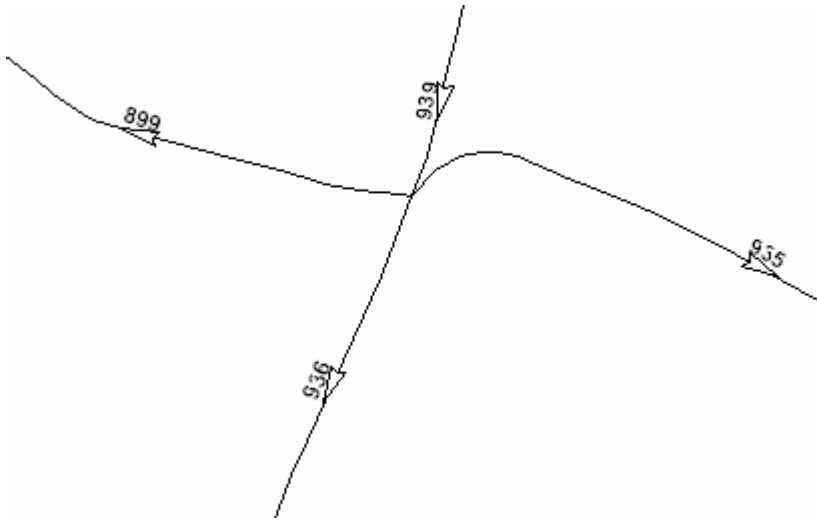
	Link Delay	Static Intersection Delay	Dynamic Intersection Delay	FHWA Guideline
<b>Number of links compared</b>	1124 (out of 2332)	1124 (out of 2332)	1124 (out of 2332)	
<b>Root mean squared error</b>	37.72	39.13	39.102	<.40
<b>Percent sum of diff</b>	1.597435%	3.01	2.31	< 5%
<b>Correlation coefficient</b>	0.901	.898	0.904	0.88

**TABLE 3: Vehicles Mile Traveled and Vehicle Hours Traveled for the Three Approaches**

	Link Delay	Static Intersection Delay	Dynamic Intersection Delay
<b>VMT</b>	428175.5	428394.4	428520.4
<b>VHT</b>	17897.6	13309.35	13500.62



**FIGURE 4 The Signalized (5-Corner) Intersection with Link IDs**



**FIGURE 5 The Unsignaled (Roosevelt/ Main Street) Intersection with Link IDs**

TABLE 4 Link Attributes for the Signalized Intersection (5-Corner)

Link ID	Attribute	Link Delay	Static Intersection Delay	Dynamic Intersection Delay
1177	Counts (NB)	965	965	965
	Counts (SB)	434	434	434
	Volume (NB)	829.91	852.37	846.01
	Volume (SB)	518.46	540.91	579.41
	Speed (NB)	13.93	13.26	13.45
	Speed (SB)	22.30	21.87	21.03
	Link Delay (NB)	.57	1.65	1.94
	Link Delay (SB)	.98	1	1.49
	FF Travel Time	.87	.87	.87
1185	Counts (EB)	278	278	278
	Counts (WB)	368	368	368
	Volume(EB)	611.94	605.67	601.65
	Volume(WB)	558	553.70	547.95
	Speed(EB)	20.24	20.40	21.85
	Speed(WB)	21.50	21.60	22.60
	Link Delay(EB)	.80	.80	.73
	Link Delay(WB)	.76	.75	.93
	FF Travel Time	.65	.65	.65
1173	Counts (NB)	622	622	622
	Counts (SB)	411	411	411
	Volume (NB)	598.25	665.46	682.91
	Volume (SB)	449.33	456.85	474.20
	Speed (NB)	20.58	18.81	20.51
	Speed (SB)	23.40	23.30	23.79
	Link Delay (NB)	.56	.61	.55
	Link Delay (SB)	.49	.50	.69
	FF Travel Time	.46	.46	.46
1174	Counts (NB)	443	443	443
	Counts (SB)	232	232	232
	Volume (NB)	480.02	502.90	504.24
	Volume (SB)	386.65	473.95	486.39
	Speed (NB)	22.96	22.58	23.47
	Speed (SB)	24.01	23.05	23.67
	Link Delay (NB)	.39	.39	.37
	Link Delay (SB)	.37	.39	.58
	FF Travel Time	.36	.36	.36
1175	Counts (EB)	599	599	599
	Counts (WB)	505	505	505
	Volume(EB)	544.63	557.67	592.07
	Volume(WB)	559.80	579.90	641.58
	Speed(EB)	21.79	21.51	22.25
	Speed(WB)	21.46	21.02	21.06
	Link Delay(EB)	.65	.66	.86
	Link Delay(WB)	.66	.67	.84
	FF Travel Time	.57	.57	.57

**TABLE 5 Link Attributes for the Unsignalized intersection (Roosevelt/ Main Street)**

<b>Link ID</b>	<b>Attribute</b>	<b>Link Delay</b>	<b>Static Intersection Delay</b>	<b>Dynamic Intersection Delay</b>
<b>939</b>	<b>Counts (NB)</b>	957	957	957
	<b>(SB)</b>	644	644	644
	<b>Volume (NB)</b>	978.91	976.09	1021.14
	<b>(SB)</b>	559.90	566.16	561.79
	<b>Speed (NB)</b>	19.69	19.83	23.88
	<b>(SB)</b>	42.93	42.65	45.45
	<b>Link Delay (NB)</b>	.72	.72	.59
	<b>(SB)</b>	.33	.33	.31
	<b>FF Travel Time</b>	.28	.28	.28
<b>936</b>	<b>Counts (EB)</b>	631	631	631
	<b>(WB)</b>	298	298	298
	<b>Volume(EB)</b>	683.05	683.83	693.9
	<b>(WB)</b>	288.1	284.58	278.27
	<b>Speed(EB)</b>	36.63	36.62	40.54
	<b>(WB)</b>	49.43	49.46	49.70
	<b>Link Delay(EB)</b>	.80	.80	.73
	<b>(WB)</b>	.76	.75	.93
	<b>FF Travel Time</b>	.59	.59	.59
<b>935</b>	<b>Counts (NB)</b>	468	468	468
	<b>(SB)</b>	566	566	566
	<b>Volume (NB)</b>	516.23	520.44	482.99
	<b>(SB)</b>	505.65	498.16	541.11
	<b>Speed (NB)</b>	31.28	31.17	33.77
	<b>(SB)</b>	31.54	31.72	33.10
	<b>Link Delay (NB)</b>	1.78	1.79	1.82
	<b>(SB)</b>	1.77	1.76	1.80
	<b>FF Travel Time</b>	1.59	1.59	1.59
<b>899</b>	<b>Counts (NB)</b>	--	--	--
	<b>(SB)</b>	--	--	--
	<b>Volume (NB)</b>	279.59	278.94	273.41
	<b>(SB)</b>	244.95	245.38	240.29
	<b>Speed (NB)</b>	34.65	34.65	34.80
	<b>(SB)</b>	34.79	34.79	34.88
	<b>Link Delay (NB)</b>	1.16	1.16	1.26
	<b>(SB)</b>	1.15	1.15	1.14
	<b>FF Travel Time</b>	1.14	1.14	1.14

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