Application of CUBE Software in Stochastic Assignment of Public Transportation

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ABSTRACT

Stochastic assignment methods are of least tolerance to actual behavior of passengers in the network. These procedures consist of Probit and Logit methods. The only method for using these methods in modeling of complicated urban networks is applying related software. Therefore it is important to get qualification in software that can use of these methods.

In this paper, application of CUBE software in stochastic assignment of public transportation was studied. Firstly, capabilities of CUBE are presented for Logit assignment. Then capabilities of CUBE in Probit method are studied and solutions for use this method is presented. Using programming functions in this software enable us to eliminate limitations of CUBE in Probit method. By representation appropriate script, this qualification is gotten. In a simple given network, both Logit and Probit methods are applied in CUBE software. Finally, results from these methods are compared.

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INTRODUCTION

Compared with car-transit assignment, public transport assignment is much more complicated. While car networks only consist of the physical network of links, nodes and may be turns, public networks consist in addition of an organizational network of routes, terminals and transfers (Nielsen et al., 1997, 1998b). Besides the problems handling this network structure in a computable manageable way, it raises a number of questions regarding the assignment of passengers:

(a) Passengers are assumed to travel on a path with minimum generalised cost which consists of four components: in-vehicle time; waiting time; walking time; and a time

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penalty for each line change. With the exception of in-vehicle time, each of the other cost components is weighted by a sensitivity coefficient which varies among travelers and is defined by a density function.

(b) Public transport networks consist often of parallel lines with the same or different frequencies. Thus, it is often a question whether two lines should be considered as different or as one line with a higher frequency? This also raises the question how to weight frequency versus in-vehicle time?

(c) Transfers and waiting times are significant factors in public transport assignment. Some passengers may choose routes in order to minimize the number of transfers, while other minimizes travel times (or something in between).

(d) Different sub-modes in public transport mode-chains have different service levels (e.g. buses versus trains). The deterministic travel times can therefore not be used in the assignment directly, but must be weighted in some kind of utility function (to generalised travel times). The distribution of different modes utilities may be different as well: Trains are, e.g. often more precise than buses and people's preferences towards buses differ more than towards trains.

(e) While perceptions of links in the car-network can be considered rather independent, choices in public transport are often dependent. This is due to the fact that public transport assignment conceptually contains elements of mode choice. As people's choices of sub-modes in a public mode-chain depend on their preferences, the choice of the next line at a terminal depends also on the preceding choice.

(f) The public network structure is very complicated. Thus, it is not sure that each passenger is aware of all feasible routes.

Public transport assignment models have increased in complexity in order to describe Passenger's route choices as detailed and correctly as possible.

One of the methods that can model these parameters is stochastic assignment. In these models, time-dependent and stochastic minimum path is generated by a specially developed branch and bound algorithm. The assignment procedure is conducted over a period in which both passenger demand and train headways are varying. Due to the stochastic nature of the assignment problem, a Monte Carlo approach is employed to solve the problem.

The notion of stochastic route choice encompasses a number of factors:

(1) Persons do not have full knowledge of the traffic network, which means they only choose rationally according to their perceived utilities.
(2) Travel times along different routes may vary from day-to-day.
(3) Different routes are often chosen for the sake of variation.
(4) Different persons may have different preferences.

The only method for using these methods in modeling of complicated urban networks is applying related software. The aim of this paper is to supply qualification for application of stochastic assignment in Cube software. The paper is organized as follows. We start by giving some description. Then, we introduce the logit assignment process in Cube software. In this section, software algorithm in use logit model is studied. Next, the problem of use the probit model in software is expressed and the solutions for use it, is given. Finally in a numerical example the results of these two ways are compared.

REGARD TO STOCHASTIC ASSIGNMENT

Stochastic network loading models are a special case of discrete choice models. To apply these models, the probability distribution function of the (perceived) travel time on each path has to be known so that the path choice probability can be calculated. This section considers two specific route-choice models. The first is based on the multinomial logit formulation and the second is based on the multinomial probit formulation.

Multinomial Logit

One of the most widely used discrete choice models is the logit model. This model, however, can be derived from the concepts of random utility and utility maximization by assuming that the random terms of each utility function are independently and identically distributed Gumbel variates. The choice probability is then given by

$$P_k^{rs} = \frac{e^{-\theta c_k^{rs}}}{\sum_l e^{-\theta c_l^{rs}}} \qquad \forall\, \mathrm{k, r, s} \tag{1}$$

Where c_k^{rs} the measured travel time and θ is is a positive parameter. There is an algorithm in Sheffi (1985, pp. 294-297) for use this probability function in assignment that has three steps as below

Step 0: *Preliminaries*:

(a) Compute the minimum travel time from node *r* to all other nodes. Determine *r(i)* for each node *i*.

(b) Compute the minimum travel time from each node *i* to node s. Determine *s(i)* for each node i.

(c) Define ϑ i as the set of downstream nodes of all links leaving node *i*.

(d) Define F_i as the set of upstream nodes of all links arriving at node i.

(e) For each link *i-,j* compute the "link likelihood", *L(i-j),* where

$$L(i \to j) = \begin{cases} e^{\theta[r(j) - r(i) - t(i \to j)]} & \text{if } \mathrm{r(i) < r(j) \text{ and } s(i) > s(j)} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Step 1: *Forward pass.* Consider nodes in ascending order of *r (i),* starting with the origin, *r*. For each node, *i,* calculate the "link weight," *w (i -- j),* for each $j \in \vartheta_i$ (i.e., for each link emanating from *i),* where

$$w(i \to j) = \begin{cases} L(i \to j) & \text{if r(i)} < \text{r(j) and s(i)} > \text{s(j)} \\ L(i \to j) \sum_{m \in F_i} w(m \to i) & \text{otherwise} \end{cases} \tag{3}$$

Step 2: *Backward pass.* Consider nodes in ascending values of *s (j)* starting with the destination, *s*. When each node, j, is considered, compute the link flow *x (i-+j)* for each $i \in F_i$ (i.e., for each link entering *j),* by following the assignment:

$$x(i \to j) = \begin{cases} q_{rs} \dfrac{w(i \to j)}{\sum_{m \in F_j} w(m \to j)} & \text{for j = s} \\ \left[\sum_{m \in \vartheta_j} x(j \to m) \right] \dfrac{w(i \to j)}{\sum_{m \in F_j} w(m \to j)} & \text{for all other links i} \to \text{j} \end{cases} \tag{4}$$

Multinomial Probit

Following many other statistical models, it may be natural to assume that the random error term of each utility is normally distributed. This is the underlying assumption of the probit model which is based on the postulate that the joint density function of these error terms is the multivariate normal (MVN) function.

None of the analytical approximation methods can be practically applied to problems involving a large number of alternative choices. The numerical integration method cannot be applied to problems involving more than four or five alternatives due to prohibitive computational costs. The successive approximations can be applied to somewhat larger problems but, again, it would involve prohibitive computational costs for problems of the size encountered in network analysis. Furthermore, the accuracy of this method deteriorates as the number of alternatives increases. Another approach to the computation of the probit choice probabilities is based on a Monte Carlo simulation procedure. The simulation method can be applied to the computation of the choice function of any discrete choice model as shown here. To describe the technique, consider a set of utility functions: $U_k = V_k + \xi_k \ \forall \ k \in Ķ$. Given the values of $V = (V_1, ..., V_k)$**,** the simulation works iteratively as follows: A vector comprising *K* random variables is drawn at every iteration from the density function of ξ_k. Denote the drawings generated in the nth iteration by $\xi_1^n, \ldots, \xi_k^n$. The perceived utility of each alternative can now be computed by adding the systematic utility, V_k, to the (drawn) random term ξ_k^n that is, $U_k^n = V_k + \xi_k^n \ \forall \ k \in Ķ$.

Next, the maximum utility alternative is recorded. This process is repeated N times. Denoting the number of times that each alternative *k* was recorded (as the maximum utility one) by N_k, the choice probability of the kth alternative, $\boldsymbol{P_k}$**,** is given by

$$P_k \cong \frac{N_k}{N} \tag{5}$$

The use of probit-based models is to overcome the problems with overlapping routes in the logit-based assignment models. Under the assumption that:

(a) Non-overlapping links are perceived independently.

(b) Links with equal mean travel resistances have the same distribution of perceived resistances.

(c) The perceived travel resistances, $c_{a(\varepsilon)}$, are normally distributed with a mean equal to the travel resistance and with a variance proportional to the resistance

$$c_{a(\varepsilon)} \in \Phi(c_a, \varepsilon.c_a), \tag{6}$$

where Φ symbolizes the normal distribution, ca is the deterministic travel resistance for link *a* and ε is the error term. In the following, the notation $c_{a(\varepsilon)}$ is used to describe a variable or expression which is simulated according to a distribution.

Daganzo and Sheffi (1977) showed, that the probability of using a certain link or route can be described by a multinomial normal distribution resulting in the Probit model. Sheffi and Powell (1981) presented an operational solution algorithm, based on Monte Carlo Simulation that was deduced from these assumptions:

Step 1: Initialization. Set the iteration number n = 1 and set the traffic flows $T_{a(0)} = 0$ for all links *a*. The (0) in $T_{a(0)}$ stands for iteration number zero (initialization).

Step 2: Update travel resistances. Sample $c_{a(\varepsilon)} \in \Phi(c_a, \varepsilon.c_a)$, for all links a using Monte Carlo simulation.

Step 3: All-or-nothing assignment. Assign the trip matrix on the network with updated $c_{a(\varepsilon)}$'s, resulting in new traffic flows, $T_{a(tmp)}$, for all links, *a* (tmp means a temporary variable).

Step 4: Step length is set. $\xi_{(n)} = 1/n$.

Step 5: Updating: $T_{a(n)} = (1-\xi_{(n)})T_{a(n-1)} + \xi_{(n)}T_{a(tmp)}$ for all links, *a*.

Step 6: Stop criteria. Stop according to a set of stop criteria, otherwise set n = n + 1 and go to step 2.

As pointed out by Van Vuren (1994), a key aspect is that the cost components at link-level are linear additive. This is achieved by relating the variance to the mean link-cost by the error term. The error term is also often referred to as proportionality constant (e.g. in Sheffi, 1985, p. 298).

A similar probit-based concept is part of the stochastic user equilibrium (SUE), where the travel resistances are flow dependent. Thus, equilibrium is reached where no traveler's perceived travel resistances can be reduced by unilaterally changing routes. SUE was suggested by Daganzo and Sheffi (1977) and operationalised by Sheffi and Powell (1982), Sheffi (1985, pp. 309-327) goes through the formulation of SUE as a mathematical programme and presents a practical solution algorithm, which is the same as for the probit model, except for steps 2 and 3:

Step 2: Update travel resistance's. Calculate $c_{a(n)} = f(c_{(0)a}, T_{a(n-1)})$ where $f(0)$ is the speed-flow curve. $c_{(0)a}$ is the travel resistance at zero traffic level (free flow).

Step 3: Assignment. Assign the trip matrix on the network with a stochastic route choice model based on the $c_{(0)a}$. Hereby the $T_{a(tmp)}$'s are modeled for all links, a.

The solution algorithm consists of two loops: An outer of steps 2-6 and the stochastic assignment model in step 3. However, good results can be obtained with only one iteration in the inner loop (Sheffi, 1985, pp. 332-335).

In public Transport, perceived costs (generalised times) can be the sun of perceived in-vehicle times and perceived waiting times

1. Perceived in-vehicle times. Each line-arc in the real network follows a number of line segments, route segments and links. However, as the distribution is additive it does not matter if this is modelled for each link or only once for the whole line-arc. Then, if the aggregated line-arcs have similar generalised costs (the proposed criteria for the aggregation) it is reasonable to do the simulation only once for the aggregated arc. If the alternative aggregation method is used, the simulation is done for each line arc instead. Different error terms may be used to describe different sub-modes (trains are e.g. generally more precise than urban buses). Buses may also be assigned different values whether they are running in urban areas and share the right of way with the other traffic (more congestion and more passengers make them less precise), have their own right of way, or they are running in rural areas (less congestion fewer passengers).
2. Perceived waiting times. Waiting times are modelled as links in the calculation graph, but they are neither conceptually nor behaviorally to be considered as links. Thus, they do not need to fulfill the reproductive property demanded of link times and can therefore follow any proper distribution. The error components for waiting times might be different for different modes.

Different coefficients and error components might also here be applied to different sub-modes. Regarding in-vehicle times, the largest coefficients will typically be on buses and the lowest on trains, as the latter are more comfortable and regular. In addition error components may be different for different sub-modes.

Waiting times are usually weighted higher than in-vehicle times. Waiting on different modes may be weighted different as well. Sitting in a heated station may as an example is considered as more comfortable than standing in the cold rain waiting for a bus.

APPLICATION OF LOGIT MODEL IN CUBE SOFTWARE

Public transport algorithm assignment in Cube is perused in this section. As will be shown, this algorithm is based on Logit models. Public transport performs the following functions in Cube:

- Public Transport (PT) Network Development;
- Route Enumeration;
- Route Evaluation;
- Skimming;
- Loading;

In network, we define parameters for software to make network e.g. link, node, lines mode, walk path, access path, stations. The main act of assignment is done in Route Enumeration (REnu) and Route Evaluation (REva). Skimming and Loading are about outputs. In following describe REnu and REva process

Route Enumeration

Route Enumeration is the process of finding one or more discrete routes between zone pairs which have some probability of being used by passengers to travel between the zones. Route Enumeration has three distinct stages:

- Finding Minimum Cost (AON) Routes;
- Establishing Connectivity between Lines;
- Enumerating Routes.

Finding Minimum Cost Routes

Minimum generalised cost routes are found between zone pairs to establish a base line cost. Each route comprises an access leg, and one or more pairs of transit and non-transit legs, the last of which is an egress leg. As transit leg bundles are used instead of transit legs, a minimum cost route may be dis-aggregated into one or more discrete routes.

The generalised cost of the route is minimized, not the number of transfers, so it is possible for the route found to have more than MAXFERS[1] transfers. However, the impact of transfers can be reflected in the generalised cost by using sensible boarding penalties. These routes enable the user to control the 'spread' of multi-routing. Two functions are provided for the calculations of spread, selected with SPREADFUNC.

Each function has two user coded keywords SPREADFACT and SPREACONST. The Route Enumeration algorithm uses the costs and number of interchanges from the minimum cost routes and the spread to determine a set of 'reasonable' routes.

Establishing Connectivity between Lines

The connectivity between lines is defined by components. They are generated for one line at a time, and define the connectivity between that line and all other lines in the system that they connect to, within constraints imposed by controls MAXFERS, EXTRAXFERS1 and EXTRAXFERS2.

Enumerating Routes

Once the components for a line have been generated, routes are enumerated for each selected origin zone connecting to the line via a valid access leg. This is done in two steps:

- The beginning of the first route is recorded; origin zone and non-transit (access) leg. The interchange points between the first two lines of the component are examined. If the transit leg used on the first line is top of the leg bundle, and the time to the next boarding point (i.e. sum of time for transit and non-transit legs) is within the spread established earlier, and the number of interchanges meet the criteria laid down by MAXFERS, EXTRAXFERS1 and EXTRAXFERS2, the route

[1] Definition of all these parameters are in CUBE VOYAGER MANUAL

is extended by the interchange. Each interchange between the two lines is similarly examined. Then the process is repeated for the second and third lines in the component and so on until component has been fully expanded.

• In the second step, all routes found above are examined to see if they terminate in zones. If they do, and the complete route between the OD pair meets the spread, interchange and destination zone selection ROUTEO J) criteria, it is output as a route between that OD pair.

Route Evaluation

Route Evaluation is the process of calculating the 'probability of use' of each of the enumerated routes between zone pairs. Some enumerated routes may be discarded because they fail to meet certain criteria, for example their probability of use is less than a user stipulated minimum, specified by CHOICECUT.

The remainder of this topic presents the Methodology of Route Evaluation, how the Cost used for it is derived and the Models applied at decision points where there are:

Walk Choices;

Transit Choices;

Alternative Alighting Choices;

The Route Evaluation process is similar to the calculation of hierarchic Logit Models, as two passes of the data are required. The first pass starts from the destination zone end of the tree and calculates the conditional probabilities of each alternative at any decision point in the tree structure. (Trips arriving at the node may proceed towards the destination along any of the alternative next-level branches. Conditional probabilities define what proportions of the trips arriving at a node proceed by each alternative branch.)

The second pass is in the reverse direction, from the origin end, and calculates the probability of choosing each discrete route. This is simply the product of all conditional probabilities along the route. A single Expected (Generalised) Cost to Destination (ECD), often referred to as Composite Cost, is calculated from any choice or decision point in a journey to the destination. This is used for calculating the probability of use of alternative routes. An attribute of the expected cost to destination is that adding new services or improving existing ones does not lead to an increase in the cost - this reflects the reality that they enhance the utility experienced by the user. At points further from the destination, where there are alternative routes, their costs are 'combined' to form a single value for the expected cost to destination at the point under consideration.

For multi-leg trips, the expected cost to destination is calculated for each leg in turn, working away from the destination. It comprises walk, transit and wait times and is calculated at decision points (where a traveler may take one of several routes forward) using the composite cost formulation.

Where the choice is between walk or alighting point alternatives, Logit Models are used. The Logit Composite Cost formulation is used to combine costs, giving a single value which represents the set of alternatives:

$$C_{Comp} = \frac{-1}{\lambda} \log\left(\sum_{alts} e^{(-\lambda ECD_{alt})} \right) \tag{7}$$

Where:

$C_{(comp)}$ is the composite cost

λ is a scale parameter which reflects the travelers sensitivity to cost differences

$ECD_{(alt)}$ is the expected (generalised) cost to destination via a particular alternative

Where the choice is between transit alternatives, the cost to destination for each alternative is calculated by adding the cost of the transit leg (including boarding and transfer penalties) and the expected cost to destination from the end of the transit leg to the destination. The values for the transit alternatives are then combined to a single value for the expected cost from the node to the destination. This is calculated as the average of the costs associated with each alternative (weighted by the probability of the alternative).

The Walk Choice Model is applied where alternative walk choices are available, This model has a logit structure:

$$P_{(\text{walk to i})} = \frac{e^{-\lambda(CWi+\alpha ECDi)}}{\sum_j e^{-\lambda(CWj+\alpha ECDj)}} \tag{8}$$

Where:

$P_{(\text{walk to i})}$ is the probability of walk to boarding stop i

λ is the scaling factor for the model (LAMBDAW)

CWi is the walk (generalised) cost to the boarding stop i

α lies in the range 0 to 1 and is specified by ALPHA. Values of 1 means that the walk and onward costs have equal weight; lower values indicate that the walk cost has more influence than onward cost in the traveller's choice. The concept incorporated here is that traveller's willingness to walk may relate to their familiarity with the network.

ECD i is the expected (generalised) cost to destination from i.

ECD j is the expected (generalised) cost to destination from j.

Two models are available to allocate passengers to the transit choices available at a stop: the Service Frequency Model (SFM) and the Service Frequency and Cost Model (SFCM).

The Service Frequency Model is applied at a stop with a basic set of transit choices, to calculate the conditional probabilities of the individual lines in proportion to their frequency. The underlying model is equivalent to travelers arriving at random without knowledge of the timetables, and taking the first reasonable service forward from the node.

$$P_{(\text{use line l})} = \frac{\text{Frequency}_{(\text{line l})}}{\sum_k \text{Frequency}_{(\text{line k})}} \tag{9}$$

where:

$P_{(\text{use line l})}$ is the probability of using line l.

Frequency$_{(\text{line } l)}$ is the frequency of the line l (in services per hour).
Frequency$_{(\text{line } k)}$ is the frequency of the line k (in services per hour).

The Service Frequency & Cost Model is an extension of the Service Frequency Model. It assumes that travellers have knowledge of the travel time to destination associated with each of the alternative routes forward, and that the traveller is less willing to use slower alternatives. A description of the process follows.

The line with lowest expected cost to destination is selected. An iterative process considers the next fastest line of those which have not yet been selected. If the line meets the validity test defined below, it is added to the set of selected lines and the process is repeated. If not, the process terminates and the set of lines already identified are used as possible routes forward towards the destination.

The validity test for each new line computes an 'excess time' as the difference between its time to destination with the average value for the lines already selected (excluding wait time at the stop). This is then compared to the expected cost of waiting (which is based on the headway of the combined services already selected, and any wait factors). When excess time has a value of zero (i.e. it is no slower than those already selected), the line will always be a valid choice. If the excess time is more than the expected cost of waiting, then the line is not valid; it is better to ignore this line and wait for the next service from the selected set.

For intermediate values, the ratio of excess time divided by cost of waiting gives a proportion of the interval between services when it is beneficial to ignore the line under consideration. This proportion is subtracted from one to give the probability of using the line when one of its services arrives at the stop. As the probability of use varies between lines, the frequency of the combined services takes account of probability of use, as follows:

$$\text{Frequency}_{(\text{combined})} = \sum_{l} P_{(\text{use } l)} \text{Frequency}_{(\text{line } l)} \tag{10}$$

where:

$P_{(\text{use } l)}$ is the probability of using line l when a service is at the stop.
Frequency$_{(\text{line } l)}$ is the frequency of the line l (in services per hour).
Frequency$_{(\text{combined})}$ is the combined frequency of a set of selected lines (in services per hour).

During loading, the proportion of demand using a particular line is given by:

$$\text{Pr}_{(\text{line } l)} = \frac{\text{P}_{(\text{use } l)} \text{Frequency}_{(\text{line } l)}}{\text{Frequency}_{(\text{combined})}} \tag{11}$$

The Alternative Alighting Model is applied when there are two or more valid alighting points for a line. This model has a logit structure similar to the Walk Choice Model:

$$P_{(\text{alight at i})} = \frac{e^{-\lambda \text{ECDi}}}{\sum_{j} e^{-\lambda \text{ECDj}}} \qquad (12)$$

where:

$P_{(\text{alight at i})}$	is the probability of alighting at stop i.
α	is the scaling factor for the model (LAMBDAA).
ECDi	is the expected (generalised) cost to destination via alternative alighting points i.
ECDj	is the expected (generalised) cost to destination via alternative alighting point j.

At transfer points between transit services, either via walk or direct, a combination of models is applied. First, the Walk Choice Model allocates demand between the actual walk alternatives and a notional one representing the transit services available at the node. Next, the Service Frequency Model allocates the transit demand to the various services at the stop.

APPLICATION OF PROBIT MODEL IN CUBE SOFTWARE

Notice to previous subjects, seeing that the main activity of cube software for public transportation is upon the logit stochastic assignment. Three main items are visible in the algorithm that present for probit stochastic assignment:

1. Sample $c_{a(\varepsilon)} \in \Phi(c_a, \varepsilon.c_a)$, for all links *a.*
2. All-or-nothing assignment
3. Updating: $T_{a(n)} = (1-\xi_{(n)})T_{a(n-1)} + \xi_{(n)}T_{a(tmp)}$ for all links, *a.*
4. Stopping test

Same as the previous section said, assignment algorithm and function in cube are based upon the logit assignment and there is no possibility to access and change them. Thus we can't do occasion number 2 in the cube. We can do only item number one from the up occasion in the cube software and items number 3&4 can done in the other database software (e.g. excel, access, FoxPro).

For implement occasion number 1 in software, i.e. spot normal distribution for time trip and random choice from it, can do in these process:

There are statements in public assignment in cube that can be used for this intention. These statements are: LINKREAD phase, PARAMETERS, GO TO, BREAKE, IF control statement. The script to do occasion number 1 is:

```
RUN PGM=PUBLIC TRANSPORT
    PARAMETERS TRANTIME= lw.GENTRSTIME
    PROCESS PHASE = LINKREAD
        A= ((LI.DISTANCE*60)/LI.SPEED)*0.5,
        B= ((LI.DISTANCE*60)/LI.SPEED)*2, Sigma=0.2,
        MU= (LI.DISTANCE*60)/LI.SPEED, U1=RAND (), U2=RAND (),
        t=A+ (B-A)*U1,
        F=1/(2*3.14*sigma)*EXP((-((t-MU)^2))/2/(SIGMA^2)),
        UU2=F/ (1/ (2*3.14*sigma))
        GOTO ARM
            U1=RAND (), U2=RAND (), t=A+ (B-A)*U1, F=1/(2*3.14*sigma)*EXP((-((t-
            MU)^2))/2/(SIGMA^2)),
            UU2=F/ (1/ (2*3.14*sigma))
        : ARM
        IF (U2<= F/ (1/ (2*3.14*sigma)))
          lw.GENTRSTIME=t
        ELSE
          BREAK
        ENDIF
    ENDPROCESS
ENDRUN
```

The keyword TRANTIME is defined by PARAMETERS statement. The PARAMETERS control statement provides global parameters for the run as a whole. And TRANTIME specifies how the base transit time for links traversed by lines is obtained.

The keyword TRANTIME is put equal to lw.GENTRSTIME variable and this variable will defined in LINKREAD phase for all the links in the network. Phase LINKREAD allows the user to compute link based information from the link Attributes of the network input with NETI. The control statements within Phase LINKREAD are executed once per link.

In the presented script, the ability of definition the normal function for links travel time and choice random through it, is provided in LINKREAD phases. Toward appropriate cover of normal function in Monte Carlo simulation, the decline and reception method is used. This method works as follows. If A & B are the upper and lower limits of a link travel time and μ is the average travel time with N amplitude and F (t) is the normal density function of link travel time, the algorithm to create appropriate cover of normal function through choice random travel time is:

1. Generate random number U1 and U2.
2. Calculate t= A+ (B-A).U1.
3. Make F (t).
4. If U2 ≥ F/N then t can be a sample otherwise go to step 1

This method is used via GOTO, IF, BREAK and RAND ( ) function in LINKREAD phase and quantification the needful parameters. Also the average value is expressed base on the LI.DISTANCE & LI.SPEED parameters that are links attributes in the network and the coefficient number 60 is used for transform the units.

Hereby, someone can apply random select through the normal distribution functions of links travel time in cube software. The next step in Monte Carlo simulation is all or nothing assignment. As expressed previously we can't use it in this software. Therefore logit function is used instead of AON assignment.

NUMERICAL EXPERIMENT

The cubetown network in the software is used to compare these two methods. Figure 1 displays this network. The logit method is practicable in normal condition. Similar to script that was shown in previous section is utilized toward use the probit method. This network was run for 1000 iteration. Then via tools that provided in cube, volume and travel time of any links of public transportation lines were storage in a database file, in per iterations. For use of this data, the values of link travel time and volume are copied from all files and store in one file. Hence, there are 1000 time and assigned volume values for each link.

Results from two manners are compared with the result from logit method. First, the final volume for each link is obtained from the average of 1000 iteration, in which there is a good amplitude distribution for link travel time, like the normal function. Second, it is obtained from the convergence criterion $\max_a\{\sigma_a^{(L)}/x_a^{(L)}\} \leq k$ (parameters are defined in equation (13) & (14)). Three answers for a link are shown in table 1.

$$\sigma_a^{(L)} = \sqrt{\frac{1}{L(L-1)}\sum_{m=1}^{L}[X_a^{(m)} - x_a^{(L)}]^2} \qquad \forall\, a \tag{13}$$

$$x_a^{(l)} = \frac{1}{l}\sum_{m=1}^{l} X_a^{(m)} \tag{14}$$

Table 1. Model Comparison

Method	Logit	Probit	
		Average of 1000 Iteration	Convergence Criterion
Volume that Assigned to Link	35	33	32.5

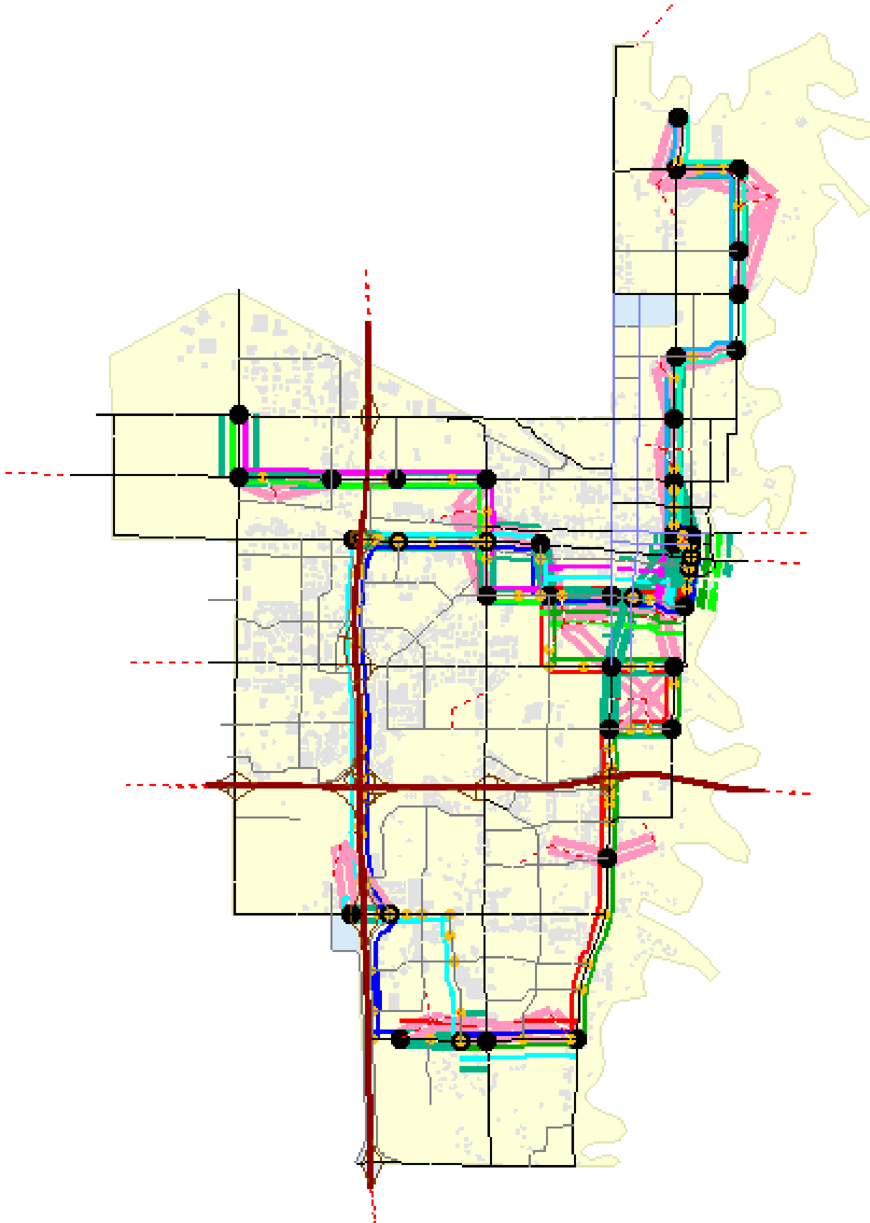

Figure 1. Test Network

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